The Project Scheduling Problem with Non-Deterministic Activities Duration: A Literature Review

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Abstract:

Purpose: The goal of this article is to provide an extensive literature review of the models and solution procedures proposed by many researchers interested on the Project Scheduling Problem with non-deterministic activities duration.

Design/methodology/approach: This paper presents an exhaustive literature review, identifying the existing models where the activities duration were taken as uncertain or random parameters. In order to get published articles since 1996, was employed the Scopus database. The articles were selected on the basis of reviews of abstracts, methodologies, and conclusions. The results were classified according to following characteristics: year of publication, mathematical representation of the activities duration, solution techniques applied, and type of problem solved.

Findings: Genetic Algorithms (GA) was pointed out as the main solution technique employed by researchers, and the Resource-Constrained Project Scheduling Problem (RCPSP) as the most studied type of problem. On the other hand, the application of new solution techniques, and the possibility of incorporating traditional methods into new PSP variants was presented as research trends.

Originality/value: This literature review contents not only a descriptive analysis of the published articles but also a statistical information section in order to examine the state of the research activity carried out in relation to the Project Scheduling Problem with non-deterministic activities duration.

Keywords: project scheduling, random duration, uncertain duration, stochastic duration, task duration

1. Introduction

The project scheduling problem (PSP) is a generic name for every problem that focus on the optimizing of the project duration, the allocation of the project resources, the estimated project costs, and the project’s cash flow, among others. In order to achieve these main goals, PSP aim to generate a sequence of activities, organized according to a decision criterion to give a proper solution to problem addressed.

Considered as an NP-hard problem (Lancaster & Ozbayrak, 2007), PSP models usually have a deterministic approach. However, some models might include uncertainty or randomness in the input parameters, corresponding to a non-deterministic direction.

This article presents an exhaustive literature review, identifying the existing models where the activities duration were taken as uncertain or random parameters. In order to get published articles since 1996, was employed the Scopus database.
The PSP with non-deterministic activities duration includes at least the following versions: Basic Project Scheduling Problem (PSP), Resource-Constrained Project Scheduling Problem (RCPSP), Resource-Constrained Project Scheduling Problem with multiple objectives (Multi-Objective RCPSP), Multi-Mode Resource-Constrained Project Scheduling Problem (MRCPSP), Resource-Constrained Multiple Project Scheduling Problem (RCMPSP), and Time/Cost Trade-off Problem (TCTP).

Solution procedures presented in literature, were classified by Brčić, Kalpic and Fertalj (2012) according to three specific approaches: a predictive strategy, a proactive strategy and a reactive strategy:

The predictive approach take the average activities duration as input data and creates a project baseline. The problem is solved as a deterministic problem.

The proactive approach takes into account the variation of the activities duration, and generate a robust baseline for the project. The robustness concept indicates that the linebase will require little changes when the project risks appear. Three types of proactive solution were identified in literature (Brčić et al., 2012): redundancy based methods, robust scheduling methods, and contingent scheduling methods.

- Redundancy based methods provide additional time for the activities in order to face the risks. The additional time can be incorporated for extending the original duration of each task or inserting buffers.
- Robust scheduling methods proposes a baseline for the project through optimization model whose objective function is based on a robustness measure. The most common robustness measure can be obtained by the weighted sum of the absolute deviation between the planned and realized activity start times (Van De Vonder, Demeulemeester, Herroelen & Leus 2006).
- Contingent scheduling methods generates more than one baseline for the project. The risks are analyzed previously and a baseline is created for each possible disruption, then there will be alternative action plans.

The reactive approach creates a strategy to re-schedule the original schedule when an unexpected event takes place. The re-schedule can be carried out by re-schedule the whole original sequence (using an optimization process) or re-schedule a little part of the network with the following strategies:

- The Right Shift Rule, where the delayed tasks should be to move toward the right consuming their slack times. If the slack time is not enough, the actions to reduce the subsequent activities duration are necessary.
- Activity crashing, where the subsequent activities duration should be reduced. This action implies an increase in the amount of resources and it generates additional costs. The Time/Cost Trade-off Problem (TCTP) allow to identify the activities that must be intervened.
- Activities overlapping, where the types of precedence should be redefined. If a project is behind schedule and the policy shows that the activities only start when its predecessors end (end-start precedence), the new policy can suggest a change in the type of precedence to the stat-start or end-end precedence.

The relationship between proactive and reactive scheduling is very strong, since a project baseline requires a reactive strategy to face the disruptions that appear during the project execution.

However, Rostami, Creemers and Leus (2017) present an alternative reactive strategy, called purely reactive strategy, on-line strategy, and also known as stochastic scheduling, one that doesn’t generate a project baseline, requiring the design of a policy or decision rule to schedule the project activities.

On Section 2, the PSP with non-deterministic activities duration is addressed, and relevant statistical information created from the literature review process is also presented. Sections 3 and 4, contains a description of the contributions and solutions techniques proposed by outstanding researchers. Section 5, some research trends are identified as a result of the statistical analysis performed, supplementary to the one reviewed on Section 2. Finally, conclusions are given in Section 6.
2. Project Scheduling Problem with Non-Deterministic Activities Duration

This section presents the PSP with non-deterministic activities duration. The documents reviewed were classified according to following characteristics: year of publication, mathematical representation of the activities duration, solution techniques applied, and type of problem solved.

2.1. According to the Year of Publication

Figure 1 shows the number of articles published between 1996 and 2017. Since 2007, the great interest of researchers on the PSP with non-deterministic activities duration is remarkable.

![Figure 1. Number of published articles per year](image)

During 1996-2017, the total number of scientific articles published was 255. However, in 2018, 3 additional articles have been identified.

2.2. According to the Mathematical Representation of the Activities Duration

On Table 1, it is observed that the most common way to represent the activities duration is through random variable. However, depiction through uncertainty measurements, such as fuzzy numbers, was employed in the 32.17% of the reviewed articles.

<table>
<thead>
<tr>
<th>Mathematical representation</th>
<th>Number of publications</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random variable</td>
<td>169</td>
<td>65.50%</td>
</tr>
<tr>
<td>Uncertainty measurement</td>
<td>83</td>
<td>32.17%</td>
</tr>
<tr>
<td>Combination: Randomness and uncertainty</td>
<td>6</td>
<td>2.33%</td>
</tr>
<tr>
<td>Total</td>
<td>258</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

Table 1. Mathematical representation of the activities duration

Additionally, a small percentage of cases, where the authors employed fuzzy numbers and random variables, is identified.

2.3. According to the Solution Techniques Applied

Table 2 indicates that traditional meta-heuristics and procedures specifically designed by researchers, were the solution techniques most applied to solve PSP with non-deterministic activities duration. Just like
meta-heuristics, the Special Procedures (SP) can generate high-quality solutions with small computational effort. SP are supported in priority rules, stage-by-stage analysis, or simple search algorithms. The exact methods, are also applied on the 21.32% of the reviewed articles, especially the Branch and Bound technique (B&B) and Dynamic Programming.

<table>
<thead>
<tr>
<th>Solution technique</th>
<th>Number of publications</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional Meta-heuristics</td>
<td>71</td>
<td>27.52%</td>
</tr>
<tr>
<td>Special Procedures (SP)</td>
<td>60</td>
<td>23.26%</td>
</tr>
<tr>
<td>Exact Methods</td>
<td>55</td>
<td>21.32%</td>
</tr>
<tr>
<td>Others algorithms</td>
<td>32</td>
<td>12.40%</td>
</tr>
<tr>
<td>Critical Chain and Buffer Management (CC/BM)</td>
<td>27</td>
<td>10.47%</td>
</tr>
<tr>
<td>Networks</td>
<td>13</td>
<td>5.04%</td>
</tr>
<tr>
<td>Total</td>
<td>258</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

Table 2. Solution techniques applied by researchers

2.4. According to the Type of Problem

The results presented on Table 3 indicate that RCPSP was the problem most frequently analyzed by researchers. This type of problem appears on the 46.12% of the published articles. However, the basic Project Scheduling Problem (PSP) was also highlighted, analyzed in the 27.91% of the reviewed articles. Others variants of the problem were reported with little frequency.

<table>
<thead>
<tr>
<th>Type of Problem</th>
<th>Number of publications</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>RCPSP</td>
<td>119</td>
<td>46.12%</td>
</tr>
<tr>
<td>PSP</td>
<td>72</td>
<td>27.91%</td>
</tr>
<tr>
<td>TCTP</td>
<td>16</td>
<td>6.20%</td>
</tr>
<tr>
<td>Multi-objective RCPSP</td>
<td>16</td>
<td>6.20%</td>
</tr>
<tr>
<td>MRCPSP</td>
<td>11</td>
<td>4.26%</td>
</tr>
<tr>
<td>RCMPSP</td>
<td>9</td>
<td>3.49%</td>
</tr>
<tr>
<td>Others</td>
<td>15</td>
<td>5.81%</td>
</tr>
<tr>
<td>Total</td>
<td>258</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

Table 3. Type of problem analyzed by researchers

3. Researchers and Relevant Articles

Articles compiled in this literature review have been presented by more than 300 researchers of different countries. Roel Leus with 12 articles and Hua Ke with 10, were the authors with the highest number of publications.

On the other hand, it is also important to mention the two most cited articles: “A Simulation-Based Process Model for Managing Complex Design Projects” (Cho & Eppinger, 2005), which has 223 citations, and presents a heuristic to schedule sequential, parallel or overlap activities; and “An investigation of buffering techniques in critical chain scheduling” (Tukel, Rom & Eksioglu, 2006), which has 111 citations, and presents two methods to determine the Feed Buffer Size under Critical Chain concept. Other highlights articles can be seen in Table 4.
Table 4. Ten most cited articles published

<table>
<thead>
<tr>
<th>Article</th>
<th>Number of citations</th>
<th>Applied techniques</th>
<th>Type of problem analyzed</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Cho &amp; Eppinger, 2005)</td>
<td>223</td>
<td>Special Procedure</td>
<td>RCPSP</td>
</tr>
<tr>
<td>(Tukel, Rom &amp; Eksioglu, 2006)</td>
<td>111</td>
<td>CC/BM</td>
<td>RCPSP</td>
</tr>
<tr>
<td>(Van De Vonder, Demeulemeester, Herroelen &amp; Leus, 2005)</td>
<td>108</td>
<td>CC/BM</td>
<td>PSP</td>
</tr>
<tr>
<td>(Long &amp; Ohsato, 2008)</td>
<td>102</td>
<td>CC/BM</td>
<td>RCPSP</td>
</tr>
<tr>
<td>(Van De Vonder, Demeulemeester &amp; Herroelen, 2008)</td>
<td>96</td>
<td>Special Procedures</td>
<td>RCPSP</td>
</tr>
<tr>
<td>(Wang, 2004)</td>
<td>92</td>
<td>Genetic Algorithm</td>
<td>RCPSP</td>
</tr>
<tr>
<td>(Golenko-Ginzburg &amp; Gonik, 1997)</td>
<td>86</td>
<td>Special Procedures</td>
<td>RCPSP</td>
</tr>
<tr>
<td>(Gutjahr, Strauss &amp; Wagner, 2000)</td>
<td>69</td>
<td>Branch and Bound</td>
<td>PSP</td>
</tr>
<tr>
<td>(Subramanian, Pekny &amp; Reklaitis, 2001)</td>
<td>67</td>
<td>Branch and Bound</td>
<td>RCMPSP</td>
</tr>
<tr>
<td>(Chtourou &amp; Haouari, 2008)</td>
<td>64</td>
<td>Special Procedure</td>
<td>RCPSP</td>
</tr>
</tbody>
</table>

3.1. Roel Leus's Contributions

Roel Leus proposed a mathematical model for the allocation of resources, which protects the project baseline against the variation on the length of the activities. The B&B algorithm was applied as solution technique (Herroelen & Leus, 2002). In 2005, Leus was co-author of a remarkable article (Van De Vonder et al., 2005), which contrasted the solutions quality of three different procedures to schedule projects: the original version of Critical Chain and Buffer Management (CC/BM), a modified version of CC/BM, and the Adapted Floating Factor method (ADFF).

Later, Leus proposed an algorithm to minimize the Stability Cost function. Current, this objective function is considered an important measure of robustness (Herroelen & Leus, 2004). In 2007 he designed an GRASP algorithm to solve the RCPSP which had greater performance than the previous algorithms (Ballestin & Leus, 2007). In 2008 he applied a backward stochastic dynamic programming recursion to maximize the Net Present Value (Creemers, Leus, De Reyck & Lambrecht, 2008; Creemers, Leus & Lambrecht, 2010). Additionally, Leus developed a new model with decision nodes to evaluate the viability of research projects (Creemers, Leus & De Reyck, 2010). In 2013, he analyzed the high uncertainty in the duration of activities and proposed two algorithms based on scenarios relaxation (Artigues, Leus & Talla Nobibon, 2013).

Recently, Leus contributed to the development of a new class of policies for project scheduling, where sequencing decisions are made in a pre-processing phase, while other decisions are made during the execution of the project (Ashtiani, Leus & Aryanazhad, 2011; Leus, Rostami & Creemers, 2015; Rostami et al., 2017).

3.2. Hua Ke's Contributions

In most of his articles, Hua Ke has utilized hybrid algorithms that integrate techniques as simulation process, schedule generation schemes (SGS), and Genetic Algorithms (GA). In 2005, a hybrid algorithm was utilized to solve a PSP with an objective function that evaluated the total cost under some completion time limits (Ke & Liu, 2005). Later, he incorporated the Fuzzy Random Variable concept to model the uncertainty on activities duration (Ke & Liu, 2007). After, in 2009 and 2010, Hua Ke tackled the Time/Cost Trade-off Problem (TCTP) (Ma & Ke, 2009; Ke, Ma, Gao & Xu, 2010), where the activities durations were assumed as fuzzy variables. In 2012, he analyzed again the previous algorithm, but in this case, the activities durations were assumed as fuzzy random variables (Ke, Ma & Ma, 2012). Additionally, Ke proposed a model called Stochastic Time-dependent Time/Cost Trade-off Problem, where the activities are executed according to original scheduling, but random delays can appear (Ke et al., 2012).

In 2015, he proposed a model to tackle the PSP, where the activities durations were assumed as fuzzy and random variables (Ke, Liu & Tian, 2015). Recently, Hua Ke explored the RCPS and developed efficient algorithms to solved it (Wang, Huang & Ke, 2015; Ma, Che, Huang & Ke, 2016). Finally, in 2017, he analyzed the Resource
Leveling Problem with uncertain durations and a deadline constraint, and proposed an algorithm that integrated a special SGS and a Distribution Estimation Algorithm (Ke & Zhao, 2017).

4. Solution Techniques

PSP with non-deterministic activities duration has been solved through the development and application of some solution techniques, such as the traditional meta-heuristics methods, special procedures (SP), exact methods, and Critical Chain and Management Buffers (CC/BM). In the following paragraphs, the main articles are mentioned and classified according to the solution technique applied.

4.1. Traditional Meta-Heuristics

Genetic algorithm (GA) was the Meta-heuristic method most frequently applied in scientific literature for the different versions of PSP with non-deterministic activities duration. However, various other Meta-heuristics as particle swarm optimization (PSO), Tabu search (TS), Bee Colony, Ant Colony, Greedy Algorithms, Simulated annealing (SA), and Distribution Estimation Algorithm (DEA), have been used as solution techniques (see Table 5).

<table>
<thead>
<tr>
<th>Meta-heuristic applied</th>
<th>Type of problem</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSP</td>
<td>RCPSP</td>
</tr>
<tr>
<td>GA</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>PSO</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>TS</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Bee colony</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Ant colony</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Greedy algorithms</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>SA</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>DEA</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Integrated algorithms</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 5. Traditional meta-heuristic applied

4.1.1. Genetic Algorithms

Hybrid algorithms were developed by Hua Ke to solve the PSP. These algorithms integrated GA with others solution techniques, as simulation process and schedule generation schemes (SGS) (Ke & Liu, 2005; Ke & Liu, 2007; Ke et al., 2012; Ke et al., 2015). Recently, Wang and Ning designed a new GA to solve an uncertain chance-constrained programming model (Wang & Ning, 2017); and on the other hand, Ji and Yao applied GA to solve an uncertain multi-objective programming model where the duration times and the resource allocation times of the activities were described as uncertain variables (Ji & Yao, 2017).

In RCPSP, the use of GA, widely exceed the number of cases reported to other PSP versions. Wang (2004) applied GA to the new product development project case; Liu, Yung, and Ip (Liu, Yung & Ip, 2007) and Liu (Liu, Zhao, Zhang & Du, 2007), found solutions to reduce the project duration. Additionally, Huang developed a procedure that integrate GA with fuzzy simulation (Huang, Ding, Wen & Cao, 2009; Wang & Huang, 2010) and another that integrated GA with a fuzzy parallel schedule generation scheme (Huang, Shou & Zhang, 2011).

In 2010, Zhao, You and Zuo (2010) designed a procedure to solve RCPSP by incorporating buffers into the project network. In 2011, Masmoudi and Haït (2011) developed an GA for Resource Leveling Problem (LRP). Later, Mogadi and Chaar (2015) designed a solution procedure that combines GA and the Forward Backward Improvement heuristic (FBI). Recently, Chen, Xiong, and Zhou (2016) applied the Resilience concept to measure the schedule’s ability to absorb possible perturbation in the project.
A Bi-objective RCPSP was solved by Zhang (2015) applying a Non-dominated Sorting Genetic Algorithm (NSGA II). This model, minimizes the project duration and maximizes the robustness of the solution. NSGA II was also used by Tabrizi and Ghaderi (2016), and Ghoddousi, Ansari and Makui (2017).

New algorithms allow to combine GA and other heuristics in order to improve their performance: Pan, Willis and Yeh (2001) combined GA and Tabu Search (TS); Wang et al. (2015) used a hybrid algorithm integrating GA and Schedule Generator Scheme (SGS); Ma et al. (2016) integrated GA with the 99-method.

4.1.2. Particle Swarm Optimization

On the other hand, PSO was another important meta-heuristic used to solve the PSP with non-deterministic activities duration. Chen, Xiao and Lu (2011) and Gan and Xu (2015) analyzed the MRCPSP and applied PSO in order to minimize the length of the project. Xu and Feng (2014) proposed an integrated model with multiples modes and multiples objectives, and Yaghoubi, Noori and Azaron (2015), solved a multi-objective continuous-time problem.

In 2014, Ma and Xu (2014) applied PSO to solve a multi-criteria multi-project RCPSP. In this article, the project owner seeks to maximize profits whereas the contractor attempts to minimize cost. Finally, PSO was utilized by Zhang to solve the RCPSP with multiple objectives or multiple modes, where the contractor is the Upper Level Decision Maker and the outsourcing partner is the Lower Level Decision Maker (Zhang, 2014; Zhang, Liu, Zhou & Chen, 2015; Zhang & Xu, 2015).

4.1.3. Integrated Algorithms

Integration of others algorithms has also been reported in the literature: Kerkhove and Vanhoucke (2017) combined Simulated Annealing (SA) and a Dedicated Algorithm; Masmoudi and Haït (2013) combined GA and Greedy Algorithm; and Kumar and Srivastava (2014), combined SA and Multi-objective GA.

4.2. Special Procedures (SP)

Special Procedures can generate high-quality solutions with small computational effort. These procedures can be supported in priority rules, simulation process, stage-by-stage analysis or simple search algorithms. Table 6 shows articles sorted according type of problem analyzed.

<table>
<thead>
<tr>
<th>Special procedure based on</th>
<th>Type of problem</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSP</td>
<td>RCPSP</td>
</tr>
<tr>
<td>Priority rules</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Simulation process</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>Stage-by-stage analysis</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Simple search algorithms</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Others</td>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 6. Special procedures applied

4.2.1. Special Procedures Based on Priority Rules

Golenko-Ginzburg and Gonik designed algorithms to select the activities that will be scheduled in the decision points located into the project network. These algorithms are designed to solve the RCPSP and to minimize the expected project duration, take into account the available resources (Golenko-Ginzburg & Gonik, 1997; Golenko-Ginzburg & Gonik, 1998). In 2007, Rabbani, Fatemi Ghomi, Jolai and Lahiği (2007) identified points decision, utilized the backward pass method to obtain the start time of each activity, and presented a new heuristic to determine the finish time.

Rabbani, Baradaran, Fatemi-Ghomi, and Hashemin (2008) tackled the RCPSP and developed a new constructive heuristic rule based on Time Criticality Index (TCI) and Resource Criticality Index (RCI). Fu, Lau and Xiao (2008),
presented the RCPSP with minimum and maximum time lags (RCPSP/max) model, and applied a Partial Order Schedule (POS) to solve the problem. Recently, Knyazeva, Bozhenyuk and Rozenberg (2015) proposed a heuristic based on priority rule to meet the project deadline.

4.2.2. Special Procedures Based on Simulation Process

Pet-Edwards built a RCPSP model and developed a special procedure supported on simulation processes (Pet-Edwards & Mollaghesemi, 1996; Fernandez, Armacost & Pet-Edwards, 1998). Golenko-Ginzburg, Gonik and Laslo (2003), proposed a simulation process to solve a network project that include both alternative deterministic decision nodes and alternative branching nodes with probabilistic outcome. Later, Golenko-Ginzburg, Gonik and Baron (2006), analyzed simultaneous projects of PERT type and some resource scheduling models.

Blaszczyk and Nowak (2009) solved the TCTP and created a new procedure based on computer simulation and interactive approach. The procedure uses simulation experiments to evaluate decision alternatives, and an interactive technique to obtain the final solution. The procedure uses stochastic dominance rules for comparing decision alternatives.

Recently, a simulation process was developed for activities crashing and to reduce the duration of the project (Subhy, Georgy & Ibrahim, 2014).

4.2.3. Special Procedures Based on Stage-by-Stage Analysis

Mizuyama (2006) formulated the PSP as a multi-stage probabilistic decision-making process. The model allow to maximize the project's output quality, and to meet the deadline. Chtourou and Haouari (2008) developed a two-stages procedure: in the first stage, the procedure uses priority rules to minimize the makespan; in the second stage, the procedure selects the best solution based on a robustness indicator.

In 2010, Hazir, Haouari and Erel (2010), presented the TCTP and designed a two-phases procedure: in the first phase the minimum required budget is determined, and in the second phase, the buffer size is maximized. Lambrechts, Demeulemeester and Herroelen (2011), designed a procedure to insert extra time due to disruptions caused by the unavailabilities of resources. The procedure includes two phases: the resources assignation and the insertion of extra time.

Leus et al. (2015), developed a procedure to find schedule policies. Initially, priority list are generated by a GRASP algorithm, and subsequently, the precedence constraints are created by a GA. Recently, in 2016, Tseng and Ko (2016) presented a two-stages procedure: in the first stage, a scenario tree is generated, and in the second stage, the worst branches are eliminated using the Expected Utility-Entropy criteria.

4.3. Exact Methods

B&B is the most representative exact technique used to solve PSP with non-deterministic activities duration (see Table 7). However, this technique has frequently been applied to project networks containing less than 60 activities, due to the high computational effort required to solve large size problems.

<table>
<thead>
<tr>
<th>Exact method applied</th>
<th>Type of problem</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSP</td>
<td>RCPSP</td>
</tr>
<tr>
<td>Branch and Bound</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>Dynamic Programming</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Stochastic programming</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Others</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 7. Exact methods applied
4.3.1. Branch and Bound Algorithm

In (Schmidt & Grossmann, 1996) a nonlinear model with a non-convex objective was transformed into a Mixed Integer Linear Program (MILP) to solve the basic PSP. Later, Gutjahr et al. (2000) developed a B&B variant to choose strategies in order to reduce the activities duration, and after, Liberatore (2008) proposed MILP models to select the project critical path. In (Jaskowski & Biruk, 2010; Jaskowski & Biruk, 2011) was designed a robust methodology based on simulation process and MILP. Then, Garaix, Artigues and Briand (2013) presented a B&B algorithm for project networks, where the activities duration were represented by an interval and the minimum float of each activity was computed over all duration scenarios.

The most recent article about basic PSP was published by Tian, Xu and Fu (2017), who take into account the time/resource trade-off to determine the best project baseline under stochastic environment.

On the other hand, B&B algorithms are also applied to solve the RCPSP. Ramat, Lente, Slimane, Tacquard and Venturini (1996) proposed a model to minimize the average project duration and ensure a low probability of conflict between resources. Leus and Herroelen (2002) used a B&B algorithm to allocate resources and protect the project baseline. Later, a model that took into account the uncertain availability of resources was presented by Li, Wang and Zeng (2010). After, Danka (2011) proposed a MILP to generate a robust baseline for the project.

Recently, Zhang, Song and Díaz (2017) proposed a new method to compute the project buffer taking into account the general average resource constraints (GARC) and the highest peak of resource constraints (HPRC).

B&B algorithm has also been employed to analyze TCTP (Ghazanfari, Yousefli, Jabal Ameli & Bozorgi-Amiri, 2009; Said & Haouari, 2015), RCMPSP (Subramanian et al., 2001), and Multi-objective Multi-mode RCPSP (Gutjahr, 2015).

4.3.2. Dynamic Programming Technique

In (Creemers et al., 2008; Creemers et al. 2010; Creemers, De Reyck & Leus, 2015) was applied Dynamic Programming to solve the basic PSP. They analyzed the activities’ risk of failure and their impact over the overall project. Later, Sobel, Szmerekovsky and Tilson (2009) presented an algorithm based on Dynamic Programming to optimize the expected present value of a project’s cash flow.

The Dynamic Programming has been also utilized to solve the RCPCP. Choi, Realff and Lee (2004), combined heuristics procedures through Dynamic Programming; Tereso, Araujo, and Elmaghraby (2004) suggested approximation schemes to reduce the computational effort; Li and Womer (2015) developed efficient algorithms support on the rollout policy; and Creemers (2014, 2015) proposed algorithms based on Stochastic Dynamic Programming and their results were evaluated using computational experiments.

4.3.3. Stochastic Programming

Chance constrained programming and Two-stage stochastic programming were techniques applied by researchers to solve the PSP with non-deterministic activities duration. In the first case, Zafra-Cabeza, Ridao and Camacho (2004) applied chance constraints to analyze risks exposures and propose a Mixed Integer Linear Problem (MILP); Bruni, Guerriero and Pinto (2009) focused on obtaining relevant information about the project makespan; and Lamas and Demeulemeester (2014) adapted the chance constrained RCPSP and created a new procedure to generate a proactive baseline.

In the second case, Klerides and Hadjicostantinou (2010, 2015) tackled the MRCPSP and proposed a two-stage stochastic integer programming to find the execution modes (stage one) and the activities scheduling (according to the activities duration obtained in the second stage). After, in 2017, Bruni, Puglia, Beraldi and Guerriero (2017) designed an algorithm based on Benders Decomposition to solve the Two-Stage RCPSP, where the resource allocation decisions were obtained in the first stage and the minimization of the worst-case makespan in the second stage.
4.4. Critical Chain Method

Critical Chain and Buffer Management (CC/BM) methods presented in this review are really improved procedures compared to the original approach. Identify and quantify key factors to determine the appropriate location of project buffers are the objectives of the CC/BM method. Table 8 shows articles sorted according type of problem analyzed.

<table>
<thead>
<tr>
<th>Type of problem</th>
<th>PSP</th>
<th>RCPSP</th>
<th>TCTP</th>
<th>Multi-objective</th>
<th>MRCPS</th>
<th>RCMPS</th>
<th>Others</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC/BM</td>
<td>7</td>
<td>19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>27</td>
</tr>
</tbody>
</table>

Table 8. Critical Chain and Buffer Management applied per type of problem

4.4.1. CC/BM Applied to PSP

Van De Vonder et al. (2005) compared three approach: the original CC/BM, a modified CC/BM, and the Adapted Floating Factor method (ADFF). After, two articles included strategies to integrate CC/BM with simulation processes. First, Yang, Fu, Li, Huang and Tao (2008), computed the buffer size of the project taking into account the network size, the uncertainty on activities duration, and the flexibility of the starting times of each activity. Second, Mansoorzadeh and Yusof (2011), identified and analyzed the critical risk factors of the projects, and evaluated their impact on the activities duration.

4.4.2. CC/BM Applied to RCPSP

The use of CC/BM to RCPSP, widely exceed the number of cases reported to other PSP versions. Methods based in Fuzzy Theory to determine the size of the buffers, was frequently applied (Zhang & Chen, 2008; Long & Ohsato, 2008). Ashtiani, Jalali, Aryanezhad and Makui (2007), modified the traditional Root Square Error Method (RSEM); Saihipal and Singh (2014), proposed a technique based on split and reassign the project buffer; and Zhang, Song and Díaz (2016) designed a method to determine the extra work (re-work), caused by lack of information between activities. Additional articles on CC/BM applied to RCPSP are available in scientific literature (Tukel et al., 2006; Fallah & Ashtiani, 2010; Roghanian, Alipour & Rezaei, 2017).

Another type of articles focus on identify new factors that affect the project buffer size. Shi and Gong (2009) analyzed the scarce resources, network complexity and the project manager's risk profile. Zhang, Cui, Bie and Chai (2011), identified a high degree of interaction between feed buffer size, service level, uncertainty on activities duration and total project duration. Liu, Chen and Peng (2012) analyzed the resource utilization factor, the network complexity and the risk aversion. Finally, Iranmanesh (2016), designed a density factor which considers scarce resources, location of the tasks on the network, work environment risks, and risks in each activity.

4.5. Network-Based Methods

Classic approaches like CPM, PERT or GERT allows model the PSP through the networks. Researchers have also used these methods to solve PSP with non-deterministic activities duration (see Table 9).

<table>
<thead>
<tr>
<th>Type of network applied</th>
<th>Type of problem</th>
<th>PSP</th>
<th>RCPSP</th>
<th>TCTP</th>
<th>Multi-objective</th>
<th>MRCPS</th>
<th>RCMPS</th>
<th>Others</th>
<th>Total</th>
</tr>
</thead>
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<td>CPM</td>
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<td></td>
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<td></td>
<td>6</td>
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<tr>
<td>PERT</td>
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<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>GERT</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

Table 9. Type of Network applied
4.5.1. CPM Networks

Lee (2005), presented a software to determine the probability of finishing the project on deadline. Later, Archer, Armacost and Pet-Armacost (2009) proposed a new technique based on CPM networks to solve the RCPSP with stochastic activities duration and activities insertion; and subsequently, Kokkaew and Chiara (2010) presented a procedure that combines the Stochastic Critical Path method with the Envelope Method.

4.5.2. PERT Networks

Zammori, Braglia and Frosolini (2009) combined fuzzy Logic with Multi-Criteria Analysis. Fortin, Zdiliski, Dubois and Fargier (2010) solved the basic PSP, and modeled the uncertainty on activities durations by intervals, and Trietsch and Baker (2012), designed a computer application that uses two graphical tools: a predictive Gantt diagram, useful for presenting stochastic analysis results, and Flags diagram, which provides signals to facilitate the project control.

4.5.3. GERT Networks

Two articles were identified: in (Gavareshki, 2004), was presented a method that requires less computation than existing fuzzy and probability GERT methods, and in (Nelson, Azaron & Aref, 2016), was proposed a procedure based on Concurrent Engineering concept.

4.6. Other Solution Algorithms

Techniques applied less frequently to solve PSP with non-deterministic activities duration have obtained good results according their authors. In this group, the following are noteworthy: Cross Entropy Method, Beam Search algorithm, Scatter Search, Discrete-Event Simulation based on Activity Scanning Method, Co-Evolutionary Algorithm based on Teaching Learning process, and the Colliding Body Optimization algorithm, among others.

5. Research Trends

Interesting research trends presented in some articles, were based on the both critical analysis of the literature review and complementary statistical analysis. The most relevant trends are presented below.

5.1. According to Solution Techniques

Genetic Algorithms (GA), Special Procedures (SP), Branch and Bound (B&B), Dynamic Programming (DP) and improved Critical Chain (CC/BM) have been and continue to be solution methods widely used to solve PSP with non-deterministic activities duration. Surely, future work will involve robust models but applying the above techniques.

On the other hand, new algorithms to solve PSP are in constant increase. These algorithms have been applied in others types of optimization problems, however, has elicited great interest between researchers, and will constitute an interesting trend.

5.2. According to Type of Problem Modeled

The information obtained in the literature review, indicate that the most traditional variants of PSP studied by researchers are both basic PSP and RCPSP. Meanwhile, TCTP, multi-mode, multi-objective and multi-projects problems, could become an emerging research topic, due to great increase in the number of published articles.

Proactive Scheduling, Reactive Scheduling, and On-line Scheduling, will continue to be the most important strategies to deal with uncertainty.

6. Conclusion

In this article a literature review on PSP with non-deterministic activities duration was carried out. This problem has become a prominent research subject, whose relevance was evidenced by the year-to-year increase in the number of publications.
Between proposals reviewed, GA stands out as the main solution technique. In addition, RCPSP was identified as the most studied problem by researchers.

Finally, based on the current research dynamics, potential axes of future work are identified. The application of new solution techniques, and the possibility of incorporating traditional methods into new PSP variants, such as TCTP, MRCPSP, Multi-Objective RCPSP, and RCMPSP, was presented as research trends.

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