An Optimization Model for Demand-Driven Distribution Resource Planning DDDRP

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Abstract:

Purpose: Demand-Driven Distribution Resource Planning (DDDRP) has recently been proposed in the literature to deal with higher supply networks complexity, shorter customer tolerance times, and inaccurate forecasts. The DDDRP requires to position inventory buffers in critical network nodes, where the inventory level in each buffer is replenished based on actual demands rather than on demand forecasts. This paper aims to identify optimal buffer positions in a distribution network driven by the DDDRP approach and to assess the performance of the DDDRP approach compared to the conventional Distribution Resource Planning (DRP) approach.

Design/methodology/approach: First, a mixed-integer non-linear model is proposed to optimize buffer positioning under supply network constraints and with the objective of minimizing supply chain holding costs. Then, a case study is investigated to validate the optimization model and to evaluate the performance of the optimized distribution network driven by the DDDRP approach, compared to the DRP approach.

Findings: Results of the considered case study demonstrate that the distribution network optimized and driven by the DDDRP approach achieves savings of 75% in terms of total holding costs and 67% in terms of inventory amounts, compared to a distribution network driven by the DRP approach.

Research limitations/implications: Results of this paper cannot be generalized since several assumptions have been considered. Thus, addressing real case studies in different industrial contexts may be of theoretical and practical interest.

Originality/value: This paper is the first to propose a mathematical model to optimize buffer positioning in a distribution network driven by the DDDRP approach.

Keywords: demand-driven distribution resource planning, distribution resource planning, inventory management, buffer positioning, optimization

To cite this article:

1. Introduction
Numerous methods and approaches have been developed to enhance supply network performance against fluctuations and to maintain customer service levels. Among these approaches, Distribution Resource Planning (DRP) is a systematic approach that effectively drives a distribution process by specifying which product, in what quantity, and in which place should be delivered to meet the demand (Rizkya, Syahputri, Sari, Siregar, Tambunan & Anizar, 2018). DRP performs well when the supply network is highly integrated (Suwanruji & Enns, 2000) since it identifies the optimal amount of lot sizes, the ordering frequency, and the amount of safety stock in each distribution node in the network (Rizkya et al., 2018). Consequently, it reduces the total cost over the supply network and upgrades customer satisfaction. However, the DRP system is a proactive approach, since the inventories are driven based on forecasting which can generate different issues, such as the bullwhip effect (Suwanruji & Enns, 2000).

Recently, a new approach, named Demand-Driven Distribution Resource Planning (DDDRP), has been proposed by Erraoui, Charkaoui and Echchatbi (2019). This approach is inspired by the Demand-Driven Material Requirement Planning (DDMRP) concepts proposed by Ptak and Smith (2019) to deal with shorter customer tolerance times, higher supply networks complexity, and inaccurate forecasts. The DDDRP improves the information flow from customers to suppliers, thanks to DDMRP buffers that should be placed in critical network nodes. As proposed by Ptak and Smith (2019), the inventory level in each DDMRP buffer is replenished based on actual demands rather than on demand forecasts. This paper is the first to propose a mathematical model to optimize DDMRP buffer positions in a demand-driven distribution network.

Next, Section 2 presents briefly the DRP and explains the concepts of the DDMRP and the DDDRP. In Section 3, a mixed-integer non-linear model is proposed to optimize buffer positioning under supply network constraints and with the objective of minimizing supply chain holding costs. Then, a case study is investigated, and results are discussed in Section 4. Section 5 concludes and provides some research perspectives.

2. Literature Review
2.1. DRP Approach
The DRP approach is a technique for replenishing inventories in distribution centers, which integrates planning and control from interconnected resources to ameliorate system implementation (Wahyuningsih & Pradana, 2018). According to the literature, many companies have taken advantage of implementing DRP since it was developed (Martin, 1992). During the 1980s-1990s, DRP along with Material Requirement Planning (MRP) and Just In Time (JIT) were taken into account as advanced management strategies for obtaining competitive advantage in the physical distribution sector in the United States (Hou, Chaudhry, Chen & Hu, 2015). The DRP has the advantage to consider a global perspective to the distribution network, in contrast to the order-point replenishment approach which focuses on minimizing the costs for each node of the network. A comparative study conducted by Suwanruji and Enns (2000) demonstrates by simulation that DRP outperforms the order-point replenishment approach when demand and replenishment times are uncertain.

The DRP is based on dependent demand logic (Watson & Polito, 2010) which, according to (Ptak & Smith, 2019), is responsible for the nervousness over the supply network. In other words, since DRP is driven by sales forecasts, small changes in downstream levels can cause significant changes for upstream levels. Dependency between the network nodes makes also delays too long to respond to the actual demand and leads to signal distortion and more bullwhip effect (small fluctuations in demands in downstream levels will increase dramatically when we move toward upstream levels).

2.2. DDMRP and DDDRP Concepts
The DDMRP approach aims to protect the inventory flow of goods or materials from oscillations by positioning inventory buffers (usually called DDMRP buffers) in network critical points (called decoupling points) in order to create independence between supply and use of material. According to Ptak and Smith (2019), the buffer positioning is a crucial step that depends on the following factors: i) the customer tolerance time, which is defined
as the time the customer would wait for being served or receiving products before referring to an alternative, ii) the sales order visibility horizon, which is the time in which the awareness of sales orders or actual relevant demand occur, iii) the external variability related to demand and supply, and iv) the complexity of the distribution network structure.

As illustrated by Figure 1, DDMRP buffers contain three zones: red, yellow, and green. Ptak and Smith (2019) provide details of how to adjust the size of each zone, based on actual demand information. One way to calculate the Top Of Green (TOG) zone is based on Equation (1).

\[
\text{TOG} = (\text{ADU} \times \text{DLT} \times \text{LTF} + \text{ADU} \times \text{DLT} \times \text{LTF} \times \text{VF}) + (\text{ADU} \times \text{DLT}) + (\text{ADU} \times \text{DLT} \times \text{LTF}) \\
= (\text{ADU} \times \text{DLT}) \times (\text{LTF} \times \text{VF} + 2 \times \text{LTF} + 1)
\]

(1)

where:

- ADU is the average daily usage, that corresponds to the average daily demand over a time frame (e.g., 14-day time frame);
- DLT is the decoupled lead time, which is a form of cumulative lead time required to deliver a product, depending on the product structure and decoupling point positions;
- LTF is the lead time factor, that takes different values depending on whether a product has high, medium, or low lead time; and
- VF is the variability factor, that takes different values depending on whether a product experiences high, medium, or low variability.

Inspired by the DDMRP approach, the DDDRP takes advantage of buffer positioning based on a dynamic point of view (Erraoui et al., 2019). Calculating buffer levels brings up to date in accordance with the average daily usage of the products consumed by customers, in contrast to DRP where the programming horizon is static, and safety stocks based on demand forecasts are used to protect against stock-outs in the supply network nodes. Thus, DDDRP can better respond to demand changes (Erraoui et al., 2019).

Demand variability is known as a principal form of variability related to distribution networks (Ptak & Smith, 2019). For example, regional warehouses have more demand variability compared to the source unit for an assumed period. However, regional warehouses can be inefficient if the inventory is higher than local demand, and consequently generate additional costs related to cross-shipping. Ptak and Smith (2019) propose three configurations to address the problems of typical distribution networks, based on decoupling points. The first configuration considers a decoupling hub located near the sourcing unit. The second configuration is a multi-hub configuration where every warehouse can be a hub (serves the customers from the same region) and a spoke (serves other regional warehouses) at the same time. The third configuration is a hybrid configuration that considers a partial hub and a spoke. Slow-moving items can be kept in the decoupling hub and fast-moving items are sent to regional centers directly from spoke (sourcing unit). Both Erraoui et al. (2019) and Ptak and Smith (2019) underline
the importance of the inventory positioning step in a demand-driven distribution network. However, they didn't propose a tool to make positioning decisions (Azzamouri, Baptiste, Dessevre & Pellerin, 2021), in contrast to this paper. The principal contributions of this paper are i) to propose a mathematical model to optimize buffer positioning in a demand-driven distribution network, and ii) to evaluate the performance of the DDDRP approach compared to the conventional DRP approach.

3. Methods
3.1. Problem Definition and Assumptions
In this paper, we consider a multi-echelon distribution network with \( I \) stages and \( J \) nodes including a manufacturing plant, and \((J-1)\) distribution nodes. Figure 2 illustrates an example with 4 stages and 22 nodes. We aim to optimize the buffer positioning in the network driven by the DDDRP approach.

We assume that each distribution node, which refers to a warehouse or a depot, is served by a single supplier. Nevertheless, each distribution node can have more than one child. The manufacturing plant is considered as an infinite capacity supply source. Products are then sent either to warehouses to be stored and maintained temporarily, or to depots to be inspected, segregated, and dispatched after orders are received from customers. We assume that the holding cost depends on the location of the distribution node in the network and that a distribution node has infinite capacity.

We assume that each customer has a normally distributed daily demand with a mean Average Daily Usage (ADU), and a standard deviation (\( \sigma \)). ADUs of different customers are considered to be independent, while ADUs for the distribution nodes are calculated based on customers’ ADU.

We set a customer tolerance time for each customer, which refers to the time by which the customer expects to receive his demand. According to APICS, customer tolerance time is “the amount of time potential customers are willing to wait for the delivery of a good or a service” (Ptak & Smith, 2019). The lead-time between the nodes is supposed to be known and includes transportation delays. Next, we propose a model that determines the optimal positions of buffers and identifies buffer levels in each distribution node based on its ADU and decoupled lead-time.

![Figure 2. A 22-node distribution network and buffer positioning](image-url)
3.2. Notation and Preliminary Concepts

In the mathematical model, we use the indices/sets, parameters, and decision variables presented respectively by Tables 1, 2, and 3.

| $i$ | Index of the distribution network stages. $i = 1, 2, \ldots$ |
| $j$ | Index of nodes including manufacturing plant, warehouses, and depots. $j = 1, 2, \ldots$ |
| $n_i$ | Number of nodes before stage $i$. |

Table 1. Indices and Sets

| $a_{jj'}$ | $= 1$ if $j'$ is a child of node $j$, 0 otherwise. |
| $L_{Tj}$ | Lead-time between node $j$ and its parent including launch and preparation time of the orders, the loading, transiting, unloading, and stocking. |
| $h_j$ | Holding cost at node $j$ per day. |
| $CTT_j$ | Customer tolerance time for customer node $j$. |
| $ADU_j$ | Demand at node $j$. |
| $v_j$ | Variability factor for node $j$. |
| $M$ | Big number. |

Table 2. Parameters

The first node is the supply source. Thus, it does not have any predecessor, which means that:

$$a_{j1} = 0 \quad \forall j$$

(2)

In general, we can say that:

$$a_{jj'} = 0 \quad \forall j > j'$$

(3)

Each node (other than node 1) has a unique parent (one predecessor), which means that:

$$\sum_{j=1}^{j-1} a_{jj'} = 1 \quad \forall j > 1$$

(4)

We assume that the actual demand of final customers is known. For each node other than final customers, we propose to calculate the ADU as the summation of all ADUs of the child nodes as presented by Equation (5).

$$ADU_j = \sum_{j=j+1}^{J} a_{jj'} * ADU_{j'} \quad \forall j < n_i$$

(5)

The variability factor $v_j$ reflects the demand variability. For final customers, $v_j$ are supposed to be known, depending on whether a product experiences high, medium, or low variability. For each node other than final customers, we propose to calculate the $v_j$ by Equation (6). Thus, for each parent node, we propose to calculate the variability factor as the summation of multiplication of ADU and variability factors overall related child nodes divided by ADU of the parent node.

$$v_j = \frac{1}{ADU_j} \left( \sum_{j=j+1}^{J} ADU_{j'} * v_{j'} \right) \quad \forall j \leq J - n_i$$

(6)
A binary variable that takes 1 if a buffer is in node \( j \), otherwise 0.

| \( \delta_j \) | Decoupled lead-time for node \( j \). |
| \( B_j \) | Mean inventory level at node \( j \). |
| \( k \) | Equal to the minimum of \( DLT_j \). |
| \( \alpha_j \) | Lead-time factor for node \( j \). |
| \( \lambda_j \) | A binary variable that guarantees that \( k \) will take the minimum of \( DLT_j \). |

Table 3. Decision variables

The binary variable \( \delta_j \) enables to specify the nodes to which the buffers are to be assigned, and to calculate more the buffer level in these nodes. It takes 1 if the buffer is assigned to a node, otherwise, it takes 0.

\( DLT_j \) is a positive variable defined as the decoupled lead-time, which can be defined as the longest cumulative coupled lead-time chain in a manufactured time's product structure (Ptak & Smith, 2019). DLT is calculated by summing all the manufacturing and purchasing lead times in that chain. The decoupled lead-time always includes the manufacturing lead-time of the parent. A decoupled lead-time in a distribution network configuration is the cumulative lead-time that depends on the position of buffers in the distribution network. For the first node, we consider \( DLT_1 = 0 \). The variable \( k \) is set as the minimum of \( DLT_j \).

The lead-time factor \( \alpha_j \) is a variable that should depend on \( DLT_j \) and should take a number between 0 and 1 (Ptak & Smith, 2019) depending on whether the node has high, medium, or low lead times. Ptak and Smith (2019) recommend a low lead-time factor for items with long lead-times and a lead-time factor close to 1 for items with short lead-time.

Equation (7) guarantees that lead-time factors \( \alpha_j \) vary between 0 and 1, that a low \( \alpha_j \) is set for a node with a long \( DLT_j \) and that a high \( \alpha_j \) is set for a node with a short \( DLT_j \). The variable \( k \) is forced to take the minimum of \( DLT_j \) by equations (8)-(10). Only a \( \lambda_j \) will take 1 (which corresponds to the lowest \( DLT_j \)) and guarantee that the variable \( k \) will take the minimum of \( DLT_j \).

\[
\alpha_j = \frac{k}{DLT_j} \quad \forall j
\]  
\[
k \leq DLT_j \quad \forall j
\]  
\[
k \geq DLT_j - M(1 - \lambda_j) \quad \forall j
\]  
\[
\sum_{j=1}^{j} \lambda_j = 1
\]

\( B_j \) is the mean inventory level at node \( j \) and can be expressed by Equation (11). If a buffer is positioned in node \( j \), \( B_j \) can be expressed as half of the Top of Green (TOG) of the inventory buffer, otherwise, it takes 0. TOG can be calculated by Equation (12), equivalent to Equation (1) proposed by Ptak and Smith (2019).

\[
B_j = \frac{1}{2} \cdot \delta_j \cdot \text{TOG}_j
\]  
\[
\text{TOG}_j = DLT_j \cdot ADU_j \cdot \left[ v_j \cdot \alpha_j + 2\alpha_j + 1 \right]
\]

As suggested by Equation (7), \( \alpha_j \) can be substituted by \( \frac{k}{DLT_j} \) in Equation (12). Thus, TOG can be expressed by Equation (13), and consequently, \( B_j \) can be obtained by Equation (14).
3.3. Mathematical Model

The mathematical model is formulated as a mixed-integer non-linear problem. The objective of the model is to optimize the buffer positions in the network driven by the DDDRP approach and to minimize the daily holding cost of the distribution network (which corresponds to the holding cost of the buffers in the distribution nodes).

The objective function presented by Equation (15) is subject to the constraints (8), (9), (10), (14), (16), and (17).

\[
\text{Minimize } Z = \sum_{j=1}^{I} B_j \times h_i
\]  

subject to:

\[
k \leq DLT_j \quad \forall j
\]  

\[
k \geq DLT_j - M(1 - \lambda_j) \quad \forall j
\]  

\[
\sum_{j=1}^{J} \lambda_j = 1
\]  

\[
B_j = \delta_j \times k \times ADU_j \times \left(\frac{\nu_j}{2} + 1\right) + \delta_j \times DLT_j \times \frac{ADU_j}{2}
\]  

\[
DLT_{j} = LT_{j'} + \sum_{j=1}^{j-1} a_{j,j'} \times (1 - \delta_j) DLT_j \quad \forall j' > 1
\]  

\[
DLT_1 = 0
\]  

\[
DLT_j \leq CTT_j \quad \forall j
\]

As explained in the previous subsection, Equations (8)-(10) guarantees that \( k \) takes the minimum of \( DLT_j \).

Equation (14) enables us to compute the buffer levels \( B_j \).

Constraints (16) enable us to compute the DLT of node \( j \), dependently whether there is a buffer or not. When a buffer is positioned in a node \( (\delta j = 1) \), DLT\( j \) is simply equal to the lead-time for node \( j \). When no buffer is positioned in a node \( (\delta j = 0) \), DLT\( j \) is the summation of DLT for its parent nodes plus the lead-time of node \( j \). For the first node, we consider DLT\( 1 \) = 0 since DLT\( j \) is less than the customer tolerance time. Constraints (17) guarantee to respect for the customer tolerance times.

3.4. Resolution Method

The structure of the objective function in this model is convex. In order to solve this convex mixed-integer non-linear problem (MINLP), we used the software GAMS (General Algebraic Modeling System designed for modeling and solving linear, non-linear, and mixed-integer optimization problems) with the solver BARON (Branch-And-Reduce Optimization Navigator designed to solve MINLPs). BARON implements deterministic global optimization algorithms of the branch-and-bound type that are guaranteed to provide global optima under fairly general assumptions. For details, see GAMS-Documentation (2020). Larger networks can be considered since
the free licence of GAMS enables us to generate and solve MINLP models that do not exceed 2500 variables and 2500 constraints.

4. Results and Discussion

Considering a 22-node distribution network (see Figure 2) driven by the DDDRP approach, we solve the optimization model with GAMS. Then, we evaluate the performance of the DDDRP approach compared to the performance of the DRP approach.

Table 4 presents the results of solving the model with GAMS. We obtain $\delta_j = 1$ only for the nodes 2, 3, 4, 5, 6, 7, 8 and 9, which means that buffers are positioned only in those nodes. With this solution, the value of the objective function which corresponds to the total daily holding cost is 333 $. The execution time for solving the problem with GAMS is acceptable (1.310 seconds).

<table>
<thead>
<tr>
<th>Indices</th>
<th>Parameters</th>
<th>Decision variables</th>
<th>Total holding cost</th>
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<tbody>
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<td>$i\ j$</td>
<td>$CTT_j$ (days)</td>
<td>$ADU_j$ (units)</td>
<td>$\sigma_j$</td>
</tr>
<tr>
<td>1 1</td>
<td>9045</td>
<td>-</td>
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<tr>
<td>2 2</td>
<td>4028</td>
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<td>2 3</td>
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<td>2 4</td>
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<td>3 5</td>
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<td>-</td>
<td>0.5</td>
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<td>3 8</td>
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<td>-</td>
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<tr>
<td>3 9</td>
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<td>-</td>
<td>0.5</td>
</tr>
<tr>
<td>3 10</td>
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<td>4 13</td>
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<td>4 19</td>
<td>517</td>
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<td>150</td>
<td>0.5</td>
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<td>863</td>
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<tr>
<td>4 22</td>
<td>781</td>
<td>270</td>
<td>0.8</td>
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</table>

$Z^* = 333.29$ 

Table 4. Results of the model applied to a 22-node distribution network and solved by GAMS
For comparison purposes, we need to evaluate the performance of the supply network if the DRP approach is used. Appendix A explains in detail how to compute inventory levels with the DRP approach. The inventory level $I_j$ of a node $j$ is computed using equation (A.7), considering an average order cost of 500$ by order and replenishment lead times of 2 or 3 days.

In Table 5, we present the total daily holding cost generated with the DDDRP approach, which depends on the buffer levels $B_j$ identified by the optimization model proposed in this paper. It corresponds to the total daily holding cost of the optimized network. Table 5 presents also the total daily holding cost generated with the DRP approach, which depends on the inventory levels $I_j$.

<table>
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<th>Indices</th>
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<td>$j$</td>
<td>$h_j$ ($/unit/day$)</td>
<td>$B_j$ (units/day)</td>
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<td>Total</td>
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<td>333.29</td>
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Table 5. Comparison of the total daily holding cost generated with the DDDRP and the DRP approaches

The total daily holding cost is 333 $ with the optimized network driven by the DDDRP approach, compared to 1,331 $ with the DRP approach (i.e., a reduction of 75%). For the total daily inventory, we obtain 62,222 units with the DDDRP approach, compared to 188,835 units with the DRP approach (i.e., a reduction of 67%). We can conclude that the distribution network optimized and driven by the DDDRP approach achieves a better performance than the distribution network driven by the DRP approach, in terms of daily inventory amounts and holding costs.
Figure 3 presents a visual comparison, for each node, between the inventory levels $I_j$ obtained with the DRP approach, and the optimal buffer levels $B_j$ generated with the DDDRP approach. As is shown in the figure, keeping inventories is not required in all nodes with the DDDRP approach (buffers are required only in nodes 2, 3, 4, 5, 6, 7, 8, and 9), while with the DRP approach, inventories are held in all nodes of the network. Besides, even if we consider node by node (in particular for nodes 2, 3, 4, 5, 6, 7, 8, and 9), the inventory level $I_j$ is always higher than the optimized buffer level $B_j$.

It is important to underline that for both approaches the levels of inventory/buffer are higher in upstream nodes than those in downstream nodes of the supply chain. But even for upstream nodes, the inventory levels with the DRP approach are still higher than the buffer levels with the DDDRP approach (we can see that from node 10, the $B_j$ are equal to zero, in contrast to the $I_j$).

5. Conclusion and Perspectives

This paper extends the literature about demand-driven distribution systems. Erraoui et al. (2019) and Ptak and Smith (2019) are the only authors who invoked the demand-driven distribution planning (DDDRP) approach. However, they don't propose tools for the inventory positioning step. This paper proposes a mixed-integer non-linear model to optimize buffer positioning in a demand-driven distribution network and evaluates the performance of the DDDRP approach, compared to the conventional DRP approach.

Results clearly demonstrate that the distribution network optimized and driven by the DDDRP approach achieves a better performance than a distribution network driven by the DRP approach, in terms of daily inventory levels and holding costs.

In this study, only holding costs through the distribution network are considered. Taking transportation costs into account increases the complexity of the problem and can be an interesting future research direction. The results of this paper cannot be generalized since several assumptions have been considered. Thus, addressing real case studies in different industrial contexts may be of theoretical and practical interest.

Declaration of Conflicting Interests

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Appendix A: Computing inventory levels with the DRP approach

To compare the performance of the DDDRP approach to the DRP approach, we need to compute inventory levels with the DRP approach. The inventory level $I$ of a specific node depends on the safety stock level $SS$ (Courtois, Martin-Bonnefous & Pillet, 2003).

First, the demand $D$ during a replenishment lead-time $L$ with the DRP approach is assumed to follow a normal distribution (Brander & Forsberg, 2006; Silver, Pyke & Peterson, 1998). Thus, $SS$ for a specific product is calculated via Equation (A.1).

\[ SS = k \cdot \sigma \cdot \sqrt{2L} \]

where $k$ is the safety factor, $\sigma$ is the standard deviation of demand during the lead-time, and $L$ is the lead-time.
SS = σ * Φ^{-1}(π) \quad (A.1)

where σ is the standard deviation of the total demand during the replenishment lead-time for the specific product, and Φ^{-1}(π) is the standard normal inverse cumulative distribution function at a π service level. According to Lee and Rim (2019), if the replenishment lead-time \( L \) and the demand \( D \) are considered independent random variables, σ can be calculated via Equation (A.2). Thus, Equation (A.1) can be replaced by Equation (A.3), where \( \sigma_D \) is defined as the standard deviation of daily demand, \( \mu_D \) is the average daily demand, and \( \sigma_L \) is the standard deviation of the replenishment lead-time \( L \).

\[
σ = \sqrt{L \sigma_D^2 + \mu_D^2 \sigma_L^2} \quad (A.2)
\]

\[
SS = \sqrt{L \sigma_D^2 + \mu_D^2 \sigma_L^2} * Φ^{-1}(π) \quad (A.3)
\]

The safety stock level (SS), previously expressed by Equation (A.3), can be calculated using Equation (A.4) if we assume that:

- the service level is \( π = 0.99 \) and the demand is following a normal distribution (\( Φ^{-1}(π) = 2.325 \));
- the lead-time is deterministic (\( \sigma_L \) can be considered as 0);
- \( \mu_D \) is the average daily demand (ADU);
- \( \sigma_D \) can be expressed as the standard deviation of the daily demand (\( \sigma_{ADU} \)) according to Mirzaee (2017).

\[
SS = 2.325 \sqrt{L \sigma_{ADU}^2} \quad (A.4)
\]

The inventory level \( I \) of a specific node can be computed by Equation (A.6) as half of the economic order quantity (EOQ) plus the safety stock level (SS). According to Courtois et al. (2003), the EOQ is calculated by Equation (A.5), where \( E \) is the order cost and \( h \) is the holding cost per unit per day. Finally, by inserting Equation (A.4) in Equation (A.6), the inventory level \( I \) can be obtained by Equation (A.7).

\[
EOQ = \sqrt{\frac{2 ADU * E}{h}} \quad (A.5)
\]

\[
I = \frac{EOQ}{2} + SS \quad (A.6)
\]

\[
I = \frac{EOQ}{2} + 2.325 \sqrt{L \sigma_{ADU}^2} \quad (A.7)
\]