Successive duopoly under moral hazard: 
Will incentive contracts persist?

Marta Fernández-Olmos¹; Jorge Rosell Martínez¹; 
Manuel Antonio Espitia Escuer¹; Luz María Marín Vinuesa²

¹University of Zaragoza (SPAIN); ²University of La Rioja (SPAIN) 
maferno@unizar.es; jrosell@unizar.es; espitia@unizar.es; luz-maria.marin@unirioja.es

Received December 2008
Accepted March 2009

Abstract: The central purpose of this paper is to examine the incentive contract as an equilibrium phenomenon. We analyse a model of vertical differentiation in which we deal with the strategic role of the competitor’s decisions in a successive duopoly. Is it better for a processor to offer an incentive contract to an upstream producer or the spot market? We determine the equilibrium of a game in which the processors simultaneously decide whether to offer an incentive contract or to continue at the spot market to acquire their input. Our results show that under successive duopoly, offering an incentive contract constitutes the unique equilibrium solution, which highlights the incentive contract persistence.

Keywords: incentive contract, moral hazard, successive duopoly, equilibrium

1 Introduction

According to principal-agent theory, incentive contracts (i.e., contracts that tie compensation to performance) are needed to elicit “effort” from agents to perform tasks that are valuable to the principal, but onerous to the agent (Milgrom & Roberts, 1992). There are many examples of incentive contracts used in practice, such as share tenancy in agriculture (e.g., Otsuka, Chuma & Hayami, 1992; Laffont
Successive duopoly under moral hazard: Will incentive contracts persist?

M. Fernández-Olmos; J. Rosell Martínez; M.A. Espitia Escuer; L. M. Marín Vinuesa

Holmstrom and Milgrom (1987) have shown that, where contracting is repeated many times and the agent has discretion in actions including the level and timing of effort, the structure of the optimal pay scheme is linear in the observed principal’s payoff. Hence, in vertical relationships with moral hazard the linear incentive contract is preferred to successive monopoly at the spot market. The reason for this is that the linear incentive contract aligns the individual incentives with their joint-surplus maximizing interests by trading off some of the risk sharing benefits for provision of incentives.

Can we expect the same result in the case of successive oligopolies? As the introduction of strategic behavior changes some relevant features of market competition, it cannot be presumed that the conclusions for successive monopoly carry over to successive oligopolies. In this paper, we focus on studying whether the above result holds for the case of successive duopoly.

Pioneering studies by Stiglitz (1974), Holmstrom (1979) and Shavell (1979) within the principal-agent framework have emphasized the role of risk sharing and incentive alignment as possible motivations for incentive contracts. Since these early contributions, the incentive contract has received more and more attention. In fact, many analysis have modelled the incentive contract in different contexts, such as monitoring (Baiman & Demski, 1980; Agrawal, 2002), tournaments (Lazear & Rosen, 1981; Holmstrom, 1982; Green & Stokey, 1983; Mookerjee 1984; Knoeber & Thurman, 1995; Rankin & Sayre, 2000, Hueth & Ligon, 2001), repeated agency contexts (Lambert, 1983; Rubinstein & Yaari, 1983; Radner, 1985; Rogerson, 1985; Spear & Srivastava, 1987; Fudenberg & Tirole, 1990; Ma, 1994; Matthews, 1995; Wickelgren, 2003), and agency models with several principal and agents (Barros & Macho-Stadler, 1998; Ray & Singh, 2001; Serfes, 2005; Dam & Pérez-Castrillo, 2006). Although all these models have provided a better understanding of the incentive contract under different settings, to the best of our knowledge there are no studies that have paid attention to the strategic implications of the principals’ choices. That is, if contracts are chosen strategically, taking into account, among other things, market competitive forces.
While it is difficult to find studies of the incentive contract in presence of market competition, there is no disputing the growing interest in vertical integration. The earliest literature in this field treated the case of successive duopoly without uncertainty. Greenhut and Ohta (1979) concluded that for a successive duopoly at each stage, producing a homogenous product, an integrated duopoly was the equilibrium structure. If the products are differentiated, Bonnano and Vickers (1988) showed that vertical separation is an equilibrium structure. Later contributions investigated the integration between Cournot oligopolists in both the upstream and downstream stages. When the number of oligopolists is equal in both industries, Greenhut and Ohta’s conclusions can be extended to successive oligopoly. However, these conclusions do not remain when the number of firms in each industry is unequal. In this last case, emergence of vertical integration rarely is the unique dominant strategy (Abiru, Nahata, Raychaudhuri & Waterson, 1998). Hence, the vertical integration as an equilibrium structure is sensitive to the specific conditions of market competition.

The purpose of this paper is to theoretically examine whether the incentive contract can emerge as an equilibrium outcome in vertical relationship with upstream and downstream competition under moral hazard. We consider a model of horizontal and vertical differentiation with two processors and two primary producers. We determine the equilibrium of a game in which the processors simultaneously decide whether to offer an incentive contract or to continue at the spot market to acquire their input. Our results show that, with successive duopolies, it is always a dominant strategy for each processor to offer an incentive contract to one producer. Thus, in equilibrium, both processors offer an incentive contract.

The rest of the paper proceeds as follows: Section 2 sets out the model. Section 3 introduces the concept of equilibrium and the equilibrium results. A summary and concluding remarks are in Section 4. All proofs are relegated to Appendix.

2 The model

We consider a model with two identical upstream producers, $U_1$ and $U_2$, who supply the essential input used by two identical downstream producers, $D_1$ and $D_2$, (“the processors” hereinafter). We suppose that one unit of input is needed to
produce one unit of output and there is no other input. Likewise, inputs from different producers will yield a final product whose quality is a weighted average of the quality of its inputs. We assume for simplicity that there are no processing costs.

The processors, risk-neutral, are quantity-setting (Cournot) competitors, producing a differentiated product. The differentiation can be vertical and horizontal. The inverse demand function for processor i’s product, \( i = D1, D2 \), is assumed to be linear:

\[
P_i = a + b_1 s_i - q_i - b_2 q_j \quad i = D1, D2 \ j \neq i
\]

where \( P_i \) is the price of the output, \( q_i \) is the quantity and \( s_i \) denotes the quality of the output of processor i. The parameter \( b_2 > 0 \) implies that the goods are substitutes.

Both producers are risk-averse. As is routinely assumed in the agency literature, we assume linear mean-variance risk preferences of constant absolute risk aversion. Each producer \( A, A=U1, U2 \) decides his quantity \( x_A \) and his effort \( e_A \) in quality. The quality of the input, \( s_A \), is specified by the following expression:

\[
s_A = \epsilon_A \mu, \quad \text{where } \mu, \text{ is a random variable normally distributed with mean 1 and variance } \sigma^2.
\]

The cost of producing the input is \( c(x, e^2) / 2 \), with \( c>0 \).

Each processor can acquire his input in the spot market (we denote by M this strategy) or by offering an incentive contract (I) to an upstream producer. We assume that the incentive contract is exclusive, that is, a processor can only contract with a producer and vice versa. Moreover, if a producer accepts the incentive contract, he can not supply his input at the spot market. Following Holmstrom and Milgrom (1991), we consider that the structure of the incentive contract is linear in the observed processor’s revenue This implies a two-part compensation scheme consisting of (i) a fixed payment, \( \alpha \), that is independent of the observed revenue, and (ii) an incentive payment that amounts to a positive share, \( \beta \), of the observed revenue.
3 The structure of the game

As we mentioned earlier, the main objective of this paper is to determine the equilibrium governance mechanism in the vertical relationship. To this end, we consider a two-stage game. In the first stage, the processors, simultaneously, decide whether to offer an incentive contract (I) or to remain at the spot market (M) (See Figure 1). They take their decisions based on the anticipated expected profits resulting from the second stage. In the second stage, the processor’s problem depends on the governance mechanism structure which results from the first stage.

\[ \pi_{i}^{g_1,g_2} \text{ processor } i \text{'s expected profit when he chooses the strategy } g_1 \text{ and the other processor chooses the strategy } g_2, g_1=\text{M,}I, g_2=\text{M,}I \]

Figure 1. "Processors’ decisions in the first stage”.

As we see in the Figure 1, with two processors, there are three possible structures of governance forms in this second stage. In the first, denoted by non incentive contract structure, both upstream producers and processors operate independently at the spot market. Producers set a price for the input, which processors buy at the spot market, transform it into output and compete in quantities in the downstream market. The producers, simultaneously, decide on their effort to
produce quality input and quantity input. In doing so, they face the derived demand for the upstream product anticipated from the decisions of the processors. In the downstream stage, the processors simultaneously decide on the quantity of the final good, taking as given the price of the upstream good they use as input and the consumer demand for the final good. Figure 2 illustrates the prices set by upstream producers to processors.

In the second structure, denoted by asymmetric incentive structure, a pair processor-producer remains at the spot market and the other pair set an incentive contract. In the incentive contract, the processor delegates the quantity and quality decisions to his contracting producer and determines the compensation scheme \((\alpha, \beta)\). The producer receives a payoff: \(\alpha + \beta y\), where \(\alpha\) and \(\beta\) are constant, \(\beta \geq 0\), and \(y\) is the processor’s revenue. The processor selects \(\alpha\) so that the producer gets only his reservation utility. We assume that the producer accepts any incentive contract that gives him a payoff at least as great as what he would get in his best alternative. In this case, if he refuses to signs up the incentive contract, he will obtain the resulting from competition in the spot market with successive duopoly. Figure 3 illustrates the asymmetric incentive contract structure: the dashed box represents the contractual relationship between a processor and a producer.
Finally, in the third structure, denoted by symmetric incentive structure, both pairs processor-producer set an incentive contract. The continuation game proceeds in the same way as the in the incentive contract in the previous structure, with the exception that the producer’s reservation utility is now different. A pair of contracts will be accepted by the producers if they can get at least the certainty equivalent resulting from the spot market in the previous structure (case ii). Figure 4 illustrates the structure under symmetric incentive contract.

Figure 3. "Competition under asymmetric incentive contract structure".

Figure 4. "Competition under symmetric incentive contract structure".
3.1 The expected profits of the structures

We are interested in characterizing the subgame-perfect Nash-equilibria. As usual, we solve the game by backward induction.

Case (i): Non incentive contract structure

We solve first the structure where both processors acquire their input at the spot market prices. The analysis is symmetrical for both processors. We denote by $x_{ia}$ the quantity of input acquired by processor $i$ from the upstream producer $A$, $A=U_1, U_2, i=D_1, D_2$. We solve the subgame by backward induction. Then, we start from period 2, in which given the input prices, $p_A$, processor $i$, chooses his quantity to maximize its expected profits $\pi_i^{MM}$:

$$\max_{q_i} \pi_i^{MM} = (a + b_i E(S_i) - q_i - b_j q_j) q_i - \sum_{A=1}^{2} p_A x_{id}, \quad i \neq j, \quad i, j = D_1, D_2, \quad A = U_1, U_2$$

where $q_i = \sum_{A=1}^{2} x_{id}$

$$E(S_i) = E\left( \frac{\sum_{A=1}^{2} s_A x_{id}}{\sum_{A=1}^{2} x_{id}} \right) = \sum_{A=1}^{2} e_A x_{id} / \sum_{A=1}^{2} x_{id}$$

which, after substitutions, gives

$$\max_{x_{id}} \pi_i^{MM} = \left( a - \sum_{A=1}^{2} x_{id} - b_i \sum_{A=1}^{2} x_{jd} \right) \sum_{A=1}^{2} x_{id} + b_i \sum_{A=1}^{2} e_A x_{id} - \sum_{A=1}^{2} p_A x_{id}, \quad (1)$$

The first order condition of this problem yields:

$$p_A = a + b_i e_A - 2 \sum_{A=1}^{2} x_{id} - b_i \sum_{A=1}^{2} x_{jd}, \quad A = U_1, U_2$$

Aggregation of (2) across the demands for producer $A$ from the processors yields:

$$p_A = a + b_i e_A - \frac{2 + b_i}{2} \sum_{A=1}^{2} x_A,$$

In the first period, each producer $A$, $A=U_1, U_2$, maximizes his certainty equivalent $CE_A^{MM}$ by solving the following problem:
where \( \rho \) is producers’ constant absolute risk aversion and \( \sigma_i^2 \) the variance of \( \mu_i \).

From the first order conditions of this problem, we get the equilibrium values of the input:

\[
x_A = \frac{2a}{3(2 + b_2)} \quad \quad e_A = \frac{3b_1(2 + b_2)}{2a(c + \rho b_2^2 \sigma_i^2)}
\]

Substituting these values in Eqs. 1 and 3 we get the certainty equivalents for each processor and producer:

\[
\pi_i^{MM} = \frac{4a^2}{9(2 + b_2)^2} \quad \quad i = D1, D2
\]

\[
CE_A^{MM} = \frac{2a^2}{9(2 + b_2)} + \frac{b_i^2}{2(c + \rho b_2^2 \sigma_i^2)} \quad \quad A = U1, U2
\]

**Case (ii): Asymmetric incentive contract structure**

Processor \( i, i = D1, D2 \) decides to offer an incentive contract to producer \( A, A = U1, U2 \), and the other processor \( j, j \neq i, j = D1, D2 \) and producer \( B, B \neq A, B = U1, U2 \) continue at the spot market. The important thing to recall is that the incentive contract means exclusivity for the processor and the producer.

In this subgame, the reservation utility is the certainty-equivalent that he would obtain at the Spot Market with two processors and two producers, that is, \( CE_A^{MM} \).

To determine the profits of each processor, we must simultaneously consider both processors’ problems to solve the reactions functions.

The processor \( i \) would maximize his expected profit by choosing \( \alpha_i \) and \( \beta_i \), subject to the incentive rationality and the incentive compatibility constraint of the producer \( A \):

\[
Max_{\alpha_i, \beta_i} = (1 - \beta_i)(a + b_i E(S_i) - q_i - b_2 q_j)q_i - \alpha_i \quad i \neq j, i = D1, D2 \quad A = U1, U2
\]
s.t. : \[ \alpha_i + \beta_i(a + b_iE(S_i) - q_i - b_2q_j)q_i - c \frac{(x_i^e)^2}{2} - \frac{\rho}{2} \beta_i^2b_i^2\text{Var}(S_i)q_i^2 \geq CE_A^{MM} \] (6)

\[ \max_{x_i, q_i} CE_A^{II} = \alpha_i + \beta_i(a + b_iE(S_i) - q_i - b_2q_j)q_i - c \frac{(x_i^e)^2}{2} - \frac{\rho}{2} \beta_i^2b_i^2\text{Var}(S_i)q_i^2 \] (7)

Since processor \( i \) only contracts with producer \( A \), it is obvious that \( q_i = x_A \) and \( E(S_i) = E(s_A) = e_A \). Similarly, the producer \( B \) is the only offerer of input, which implies \( q_j = x_B \) and \( E(S_j) = E(s_B) = e_B \).

The problem optimization in equation (5) and (7) can be solved sequentially. First, we begin with equation 7, which, after substitutions, gives:

\[ \max_{x_i, q_i} CE_A^{II} = \alpha_i + \beta_i(a + b_iE(S_i) - x_A - b_2x_B)x_A - c \frac{(x_i^e)^2}{2} - \frac{\rho}{2} \beta_i^2b_i^2\text{Var}(S_i)q_i^2 \]

First order conditions yield the following reaction functions in quantity and quality:

\[ a - 2x_A - b_2x_B = 0 \] (8a)

\[ e_A = \frac{\beta_ib_i}{x_A(c + \rho\beta_i^2b_i^2\sigma_i^2)} \] (8b)

Paralelally, in the spot market we proceed in the same manner as in the earlier case.

Proceeding through backward induction, the processor \( j \) solves the following problem:

\[ \max_{x_j} \pi_j^{MI} = (a + b_iE(S_j) - q_j - b_2q_j)q_j - p_Bx_B \]

which, after substitutions, gives

\[ \max_{x_j} \pi_j^{MI} = (a + b_iE(S_j) - x_B - b_2x_A)x_B - p_Bx_B \]

Optimization of this equation yields:

\[ p_B = a + b_iE_B - 2x_B - b_2x_A \]

with \( E(S_j) = E(s_B) = E(e_B\mu_\epsilon) \)

and the producer \( B \) decides \( x_B \) and \( e_B \) to:
From the first order conditions, we obtain:

\[ a - 4x_B - b_2x_A = 0 \]  

(9a)

\[ e_B = \frac{b_1}{x_B(c + \rho b_i^2 \sigma_s^2)} \]  

(9b)

Solving the equations 8a and 9a simultaneously we get

\[ x_A = \frac{a(4 - b_2)}{8 - b_2^2} \quad e_A = \frac{\beta_i b_1(8 - b_2^2)}{a(4 - b_2)(c + \rho b_i^2 b_i^2 \sigma_s^2)} \]  

(10a)

\[ x_B = \frac{a(2 - b_2)}{8 - b_2^2} \quad e_B = \frac{b_1(8 - b_2^2)}{a(2 - b_2)(c + \rho b_i^2 \sigma_s^2)} \]  

(10b)

Constraint 6, which is binding, implies

\[ \alpha_i = -\beta_i(a + b_i E(S_i) - q_i - b_2 q_j)q_i + c \frac{(x_Se_S)^2}{2} + \frac{\rho}{2} \beta_i^2 b_i^2 \text{Var}(S_i)q_i^2 - CE^U_M \]  

(11)

Finally, substituting (11) and (10a) into (5) and maximizing with respect to \( \beta_i \), it may be shown that

\[ c - c\beta_i - \rho \beta_i^2 b_i^2 \sigma_s^2 = 0 \]

Finding the value of the incentive parameter, we obtain:

\[ \beta_i = \frac{-c + \sqrt{c^2 + 4c\rho b_i^2 \sigma_s^2}}{2\rho b_i^2 \sigma_s^2} \]

\[ i = D1, D2 \]

\[ A = U1, U2 \]

We omit the other solution of the square root because it has not economic sense.

As we expected, the properties of the incentive parameter are the standard in the agency literature. That is, \( 0 \leq \beta_i \leq 1 \) and \( \frac{\partial \beta_i}{\partial (\rho b_i^2 \sigma_s^2)} < 0 \).

Now, the processors expected profits and producers certainty equivalents in each mechanism can be derived:
Successive duopoly under moral hazard: Will incentive contracts persist?

M. Fernández-Olmos; J. Rosell Martínez; M.A. Espitia Escuer; L. M. Marín Vinuesa

Case (iii): Symmetric incentive structure

Consider now the case when both processors offer an incentive contract, that is processor \( i, i=D_1, D_2 \) offers an incentive contract to producer \( A, A=U_1, U_2 \). We first determine the producer \( A \)'s reservation utility, \( A=U_1, U_2 \), which is the certainty equivalent from the producer at the spot market in the previous structure:

\[
CE^I_A = \frac{2a^2(2-b_2)^2}{(8-b_2)^2} + \frac{b_1^2}{2(c+\rho b_1^2\sigma_s^2)}
\]

with \( \beta_i = -\frac{c+\sqrt{c^2+4c\rho b_1^2\sigma_s^2}}{2\rho b_1^2\sigma_s^2} \)

\( i=D_1, D_2 \) \( j \neq i \) \( j=D_1, D_2 \) \( A=U_1, U_2 \) \( A\neq B \) \( B=U_1, U_2 \)

Since processor \( i \) only contracts with producer \( A \), it is obvious that \( q_i = x_A \) and \( S_i = S_A = e_A \mu_r \). To determine the expected profits of each processor, we proceed in the same manner as in the incentive contract of the case (ii). First, we solve the incentive rationality constraint of the producer \( A \):

\[
\max_{x_A, e_A} CE^I_A = \alpha_i + \beta_i (a + b_i E(S_i) - q_i - b_2 q_j) q_i - c \frac{(x_A e_A)^2}{2} - \frac{\rho}{2} \beta_i^2 b_i^2 \text{Var}(S_i) q_i^2
\]

which, after substitutions, gives

\[
\max_{x_A, e_A} CE^I_A = \alpha_i + \beta_i (a + b_i e_A - x_i - b_2 x_j) x_i - c \frac{(x_A e_A)^2}{2} - \frac{\rho}{2} \beta_i^2 b_i^2 e_A^2 \sigma_s^2 x_i^2
\]

The optimal solutions to this problem are:

\[
\begin{align*}
x_A &= \frac{a}{2 + b_2} \\
e_A &= \frac{(2 + b_2)\beta_i b_i}{c_A + \rho b_1^2 \sigma_s^2}
\end{align*}
\]

(12)

substituting these values of \( x_A \) and \( e_A \) in producer \( A \)'s incentive compatibility constraint, the value of \( \alpha \) is obtained. Finally, we substitute all these values in...
producer i’s problem and maximize with respect to \( \beta_i \), obtaining the same value of \( \beta_i \) than in the case (ii). Then, it is easily verified that the processor i’s expected profit is

\[
\pi_i^{III} = \frac{a^2}{(2 + b_i)^2} - \frac{2a^2(2 - b_i)^2}{(8 - b_i^2)^2} + \frac{\beta_i b_i^2}{c + \rho b_i^2 \beta_i^2 \sigma_i^2} - \frac{\beta_i^2 b_i^2}{2(c + \rho b_i^2 \beta_i^2 \sigma_i^2)} = \frac{b_i^2}{2(c + \rho b_i^2 \sigma_i^2)}
\]

The game outcomes are summarized in Table 1.

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Output quantity</th>
<th>Expected quality output</th>
<th>Processor expected profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M,M)</td>
<td>( q_i^{MM} = \frac{2a}{2(2 + b_i)} )</td>
<td>( E(S_i^{MM}) = \frac{3b_i(2 + b_i)}{2a(c + \rho b_i^2 \sigma_i^2)} )</td>
<td>( \pi_i^{MM} = \frac{4a^2}{9(2 + b_i)^2} )</td>
</tr>
<tr>
<td>(M,I)</td>
<td>( q_i^{MI} = \frac{a(2 - b_i)}{8 - b_i^2} )</td>
<td>( E(S_i^{MI}) = \frac{b_i(8 - b_i^2)}{a(2 - b_i)(c + \rho b_i^2 \sigma_i^2)} )</td>
<td>( \pi_i^{MI} = \frac{a^2(2 - b_i)^2}{(8 - b_i^2)^2} )</td>
</tr>
<tr>
<td>(I,M)</td>
<td>( q_i^{IM} = \frac{a(4 - b_i)}{8 - b_i^2} )</td>
<td>( E(S_i^{IM}) = \frac{b_i(8 - b_i^2)}{a(4 - b_i)(c + \rho b_i^2 \sigma_i^2)} )</td>
<td>( \pi_i^{IM} = \frac{a^2(4 - b_i)^2}{(8 - b_i^2)^2} - \frac{a^2(2 - b_i)^2}{9(2 + b_i)^2} + \psi )</td>
</tr>
<tr>
<td>(I,I)</td>
<td>( q_i^{II} = \frac{a}{2 + b_i} )</td>
<td>( E(S_i^{II}) = \frac{\beta_i b_i(2 + b_i)}{a(2 + b_i)(c + \rho b_i^2 \beta_i^2 \beta_i^2 \sigma_i^2)} )</td>
<td>( \pi_i^{II} = \frac{a^2(2 - b_i)^2}{(8 - b_i^2)^2} + \psi )</td>
</tr>
</tbody>
</table>

where \( \beta_i \) is defined as

\[
\beta_i = \frac{c + \sqrt{c^2 + 4c \rho b_i^2 \sigma_i^2}}{2 \rho b_i^2 \beta_i \sigma_i^2}
\]

where \( \psi \) is defined as

\[
\psi = \frac{\beta_i b_i^2}{c + \rho b_i^2 \sigma_i^2} - \frac{\beta_i^2 b_i^2}{2(c + \rho b_i^2 \sigma_i^2)} = \frac{b_i^2}{2(c + \rho b_i^2 \sigma_i^2)}
\]

Table 1. "Game outcomes".

### 3.2 The equilibrium of the game

Having determined the processor’s expected profits in each structure, we proceed now to find their equilibrium strategies. The payoffs matrix is summarized in Figure 5.

Since the two processors are symmetric, the equilibrium profits of these two processors are equal, i.e., \( \pi_A^{MM} = \pi_B^{MM} \), \( \pi_A^{II} = \pi_B^{II} \), \( \pi_A^{IM} = \pi_B^{IM} \), \( \pi_A^{MI} = \pi_B^{MI} \).
Proposition: With successive duopolies, the unique equilibrium of the game is (I, I), that is, each processor offers one incentive contract to one upstream producer.

Proof: See Appendix 1

In other words, a processor could increase his expected profits if he offers an incentive contract to a producer, no matter neither the level of the risk premium is nor the strategy the other processor chooses.

Given that there exists a direct link between processor’s profit and quantity and quality produced, we already have an intuition concerning this result.

Comparing the levels of quantity from table 1, it is easy to see that

\[ q_{i}^{IM} > q_{i}^{MM} \]
\[ q_{i}^{II} > q_{i}^{MI}, \quad i = D1, D2 \]

We find that when the processor chooses the incentive contract, his level of output is larger than the level he could produce when he buys the input at the spot market, no matter what the other processor chooses. As the quantity affects the risk premium borne by the producer, it is reasonable that the incentive contract induces more quantity than the spot market.

Comparing the levels of expected quality from table 1, we have that

\[ E(S_{i}^{IM}) > E(S_{i}^{MM}) \quad \text{if} \quad \rho b_{1}^{2} \sigma_{x}^{2} > 2c \varphi_{1} \quad \text{with} \quad \varphi_{1} = \frac{3(4 - b_{2})(2 + b_{2})}{2(8 - b_{2}^{2})} \]
\[ E(S_{i}^{II}) > E(S_{i}^{MI}) \quad \text{if} \quad \rho b_{2}^{2} \sigma_{x}^{2} > 2c \varphi_{2} \quad \text{with} \quad \varphi_{2} = \frac{8 - b_{2}^{2}}{4 - b_{2}^{2}} \]
As the agency literature predicts, as the level of risk premium increases (it can be caused by uncertainty, $\sigma^2$, risk aversion, $\rho$, and the importance of quality in the price, $b_2$), it is more likely that the expected quality for a processor in the incentive contract will be greater than in the spot market, no matter what the other processor chooses. These results suggest that although the expected quality provided by the incentive contract is smaller than in the spot market under determined conditions (i.e., low levels of risk premium), the greater level of quantity obtained compensates this reduction in quality, obtaining always a greater expected profit than at the spot market.

On balance, although the degree of competitiveness has changed from successive monopoly into successive duopoly, the symmetric incentive structure emerges as an unique equilibrium. Then, the standard result that the incentive contract persists holds under a market structure with two upstream and two downstream producers, in which the strategic effects do not affect the equilibrium structure. This absence of strategic effect is not unlike that found in a vertical integration related framework by Greenhut and Ohta (1979), when studying the equilibrium incentives to integrate vertically when successive monopoly would be the alternative. They find that an integrated duopoly is also the equilibrium when successive monopoly would be the alternative.

### 4 Conclusions

By taking into account the competitor’s decisions, we take a further step towards understanding the incentive contract choice in vertical relationships with moral hazard.

We have presented a model of vertical relationship in which upstream producers sell differentiated inputs and downstream producers process and sell them to consumers, and we have analysed the mechanisms each processor chooses to obtain the input needed to produce their goods.

To the best of our knowledge, this work is the first to address the question of incentive contract versus spot market in presence of successive duopoly. In particular, we consider a two-stage game. In the first stage, processors decide simultaneously whether or not to set an incentive contract (versus spot market). The second stage is the stage in which input producers choose their levels of
quantity and quality based on the industry structure developed in the first stage. With the help of the subgame-perfect Nash-equilibria, we conducted the study of the equilibrium structures.

This paper shows that, under successive duopoly, offering an incentive contract constitutes the unique equilibrium solution, which highlights the incentive contract persistence. This result is consistent with the evidence concerning vertical integration.

The exercise throws some light on the relative importance of analysing the quantity/quality trade-off and successive duopoly in the analysis of the optimality in market versus incentive contract choice. We can not conclude from our results that the mechanism that provides a greater level of quality is always the most efficient mechanism.

In the event that managerial policy is geared towards increasing the quality in the inputs markets, the policy makers could strive to incentive growers to produce quality by regulating the specified maximum quantity. However, the situation is complicated by the fact that in order to induce input producers to produce quality, they must be offered a compensation for the increase in cost associated with it. And if this compensation for cost increases is not associated with higher spot prices, it is difficult that input producers can receive it. The alternative, and more realistic, strategy for primary producers and processors is to think how to make that consumers perceive the quality and are willing to pay more for it. Creating a Quality Certified Brand could be a possibility given that it provides consumers with a better understanding of input quality.

This study has important limitations that imply caution in generalizing the findings. As for the number of processors considered, our results hinge on the fact there are only two upstream and downstream competitors. On the basis of previous work, it seems reasonable to conjecture that the possibility of including more processors at the spot market will make the spot market more favorable. In this case, although processors are identical, our intuition is that asymmetric incentive structures could emerge as equilibrium structures, which would be consistent with the contractual evidence. Then, it could be an interesting topic for future research to extend the model to include multiple operators.
Likewise, our results depend on the way we model the market interaction between the processors with incentive contract and the processors at the spot market. We must emphasize that the possibility of processors with an incentive contract buying at the spot market is not considered in this paper. However, processors might choose to purchase their inputs from independent upstream producers for strategic reasons, for example, to raise the rivals’ input cost. Then, another interesting extension would be to allow processors with an incentive contract to freely trade with independent upstream producers and analyse if the raising-rivals’ costs strategy influences the nature of incentive contract equilibria.

One limitation of this analysis is that all processors and primary producers are assumed identical. On the basis of previous work, for example Hendrikse (2007), it seems reasonable to conjecture that the possibility of including heterogeneous participants would not influence our qualitative results. However, the analytical difficulties associated with this issue would increase considerably. This limitation will be considered in future research efforts.

Appendix

Proof of Proposition: We need to show that the pair of strategies (I, I) is the unique equilibrium of the game.

The pair of strategies (I, I) will be an equilibrium if and only if inequality (A1) and (A2) hold:

\((A1) \quad \pi_i^{IM} > \pi_i^{MM} \quad i=D1,D2\)
\[(A2) \quad \pi_i^{II} > \pi_i^{MI}\]

Proof of inequality A1

Replacing the values of \(\pi_i^{IM}\) and \(\pi_i^{MM}\) (see table 1) we have

\[\pi_i^{IM} - \pi_i^{MM} = \frac{a^2(4 - b_2)^2}{(8 - b_2)^2} - \frac{2a^2}{9(2 + b_2)} - \frac{4a^2}{9(2 + b_2)^2} + \psi\]

where \(\psi\) is defined as

\[\psi = \frac{\beta_1 b_1^2}{c + \rho b_1^2 \beta_1^2 \sigma_1^2} - \frac{\beta_1^2 b_1^2}{2(c + \rho b_1^2 \beta_1^2 \sigma_1^2)} - \frac{b_1^2}{2(c + \rho b_1^2 \sigma_1^2)}\]

and
To verify that this expression is positive, we first show that \( \psi > 0 \).

Numerical methods show that

\[
\frac{\beta_i b_i^2}{c + \rho b_i^2 \beta_i^2 \sigma_s^2} - \frac{\beta_i^2 b_i^2}{2(c + \rho b_i^2 \beta_i^2 \sigma_s^2)} - \frac{b_i^2}{2(c + \rho b_i^2 \sigma_s^2)} > 0
\]

if and only if \( c(\rho b_i^2 \sigma_s^2)^4 + 4(\rho b_i^2 \sigma_s^2)^5 > 0 \) which is always true.

Finally, numerical methods show that

\[
\frac{a^2(4-b_2)^2}{(8-b_2)^2} - \frac{2a^2}{9(2+b_2)} - \frac{4a^2}{9(2+b_2)^2} > 0
\]

if and only if \(-2b_2^5 + b_2^4 - 4b_2^3 + 20b_2^2 + 160b_2 + 64 > 0\)

Since \( 0 \leq b_2 \leq 1 \), this inequality is always true. Therefore, inequality A1 holds.

**Proof of inequality A2**

Replacing the values of \( \pi_i^{II} \) and \( \pi_i^{MI} \) (see table 1) we have

\[
\pi_i^{II} - \pi_i^{MI} = \frac{a^2}{(2+b_2)} - \frac{2a^2}{(8-b_2)^2} + \psi
\]

where \( \psi \) is defined as

\[
\psi = \frac{\beta_i b_i^2}{c + \rho b_i^2 \beta_i^2 \sigma_s^2} - \frac{\beta_i^2 b_i^2}{2(c + \rho b_i^2 \beta_i^2 \sigma_s^2)} - \frac{b_i^2}{2(c + \rho b_i^2 \sigma_s^2)}
\]

and

\[
\beta_i = \frac{-c + \sqrt{c^2 + 4\rho b_i^2 \sigma_s^2}}{2\rho b_i^2 \sigma_s^2}
\]
The identification of the sign of \( \pi_i^H - \pi_i^M \) is straightforward. From the previous proof, we know that \( \Psi > 0 \). Moreover, it is easy to verify that

\[
\frac{a^2}{(2 + b_2)^2} - \frac{2a^2(2 - b_2)^2}{(8 - b_2)^2} - \frac{a^2(2 - b_2)^2}{(8 - b_2)^2} > 0
\]

if and only if \(-2b_2^4 + 8b_2^2 + 16 > 0\)

Since \( 0 \leq b_2 \leq 1 \), this inequality is always true. Thus, inequality A2 holds.

This completes the proof of proposition.

References


