The equilibrium results analysis in a competing supply chains with consumer returns

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Abstract:

Purpose: The purpose of this paper is to examine the optimal pricing and ordering strategy of two competing chains with customer returns in Bertrand-Nash game and Stackelberg game, and to investigate in what cases the manufacturer can make more profits from customer returns policy.

Design/methodology/approach: We build the customer returns model in the competing chains and compare the equilibrium results in Bertrand-Nash game with that in Stackelberg game.

Findings: The main contribution of the present study is the manufacturer can benefit from customer returns policy, only if customer return rate is sufficiently low in two games. In addition, the optimal price, order quantity and profits in Stackelberg game are affected more seriously by customer returns than that in Bertrand-Nash game.

Originality/value: The impact of customer returns on the competing chain is initially considered, where the demand is related with the retail price and return price.

Keywords: competing supply chain, consumer returns, Bertrand-Nash game, Stackelberg game
1. Introduction

Accepting customer returns has been an important strategy for retailers to attract customers and stimulate demand, in the increasingly competitive market environment. Customer returns policies can enhance customers’ confidence in purchasing goods, stimulating the demand and possibly increasing the retailer’s market share. However, it is also bound to increase the retailers’ and the manufacturers’ processing cost, or may even devaluate the goods and delay sales if the returned goods are resold after some processing. It has been reported that the value of products returned in the United States has exceeded $100 billion per year (Stock et al. 2002). Hence, from an operational standpoint, a natural question emerges: how should two effects of customer returns be traded off to yield greater profits?

This paper is related with literatures on customer returns policy. Customer returns impact the decision-makers’ pricing and ordering strategies. In general, they can use full refund policy, which is a 100% money-back-guarantee (MBG) offered to ensure consumer satisfaction, or offer partial refund policy for customers. Mukhopadhyay and Setoputro (2004) study the optimal retail price and return price of reverse logistic in e-business. William and Gerstner (2006) report that, in their limited sample, 87% of the stores offer full money back guarantee within a “return period”. Chen and Bell (2009) address the simultaneous determination of price and inventory replenishment when customers return product to the firm. They examine the cases when the quantity of returned product was a function of both the quantity sold and the price, in single and multi-period problems, with and without uncertainty in demand. Su (2009) studies the impact of full returns policies and partial returns policies on supply chain performance, and proposes coordination strategies. He demonstrates that full returns is excessively generous and thus fails to optimize supply chain performance.

Competition has become a hot topic in the area of supply chain management. The research of this paper is related with competing supply chains. At present, most studies concentrate on two supply chains for simplicity. There are several factors which cause the competition, such as the quantity of product, price, quality and warranty period and so on. Choi (1991) investigates two manufacturers facing the same retailers competed on price. Bernstein and Fedegruen (2004) examine a supply chain system competing on price and fulfill rate, and demonstrate the existence of Nash equilibrium. Boyaci and Gallego (2004) consider three competition scenarios between the supply chains, including the uncoordinated scenario, the coordinated scenario and the hybrid scenario. They find that coordination is a dominant strategy for both supply chains, but as in the prisoner’s dilemma, both supply chains are often worse off under the coordinated scenario relative to the uncoordinated scenario. Moorthy (1988) studies a model with two identical firms competing on product quality and price, and suggests that cannibalization has different effects on product strategy than competition. Then, Banker (1998) examines the quality and price competition of two supply chains. Recently,
Chen (2012) investigates the optimal decision of the manufacturer as the Stackelberg leader and two retailers as the followers when the demand is dependent on warranty period.

In the competitive market, customers choose which one of supply chains depending on the price to a certain extent. Therefore, the market demand is related with price. In general, the market demand decreases in the retail price. Considering the customer returns, Mukhopadhyay and Setoputro (2005) investigates a single supply chain, where the market demand is seen as a linear function of retail price and return price, and obtain the optimal pricing and return policy to make the profits of the supply chain maximize. However, they ignore the competition environment. So, how will customer returns affect the optimal pricing, ordering and supply chain performance in the competitive environment? To address the question, we consider the demand is related with the retail price and return price of both the supply chain itself and the competing supply chain. Ofek et al. (2011) investigate the impact of product returns on the strategies of multi-channel retailers in duopoly settings. However, we emphasis on this impact in different games (Bertrand-Nash and Stackelberg game).

Bertrand-Nash game and Stackelberg game in Game theory have been commonly applied in supply chain management in recent years. Bertrand-Nash game refers to the situation where two firms compete with each other on price and act simultaneously, and then each one chooses the price conditional on the other firm’s price decision. The Stackelberg game refers to the situation where one firm as a leader acts firstly, then the other firm as a follower set price in response to the leader’s price. In this paper, we study how the change of return price impact on the competing supply chain under the conditions of Bertrand-Nash game and Stackelberg game.

This paper is different with the above literature. Firstly, we initially consider that the demand is related with the retail price and return price of both the supply chain itself and the competing chain. Secondly, the return price considered in this paper is not limited to the full refund, but it can change from 0 to the retail price.

The remainder of this paper is as follows. In the next section, we state the related problem and build the consumer returns model in the competing chains. Then, we give and analyze equilibrium results in Bertrand-Nash game and Stackelberg game respectively, in Section 3. In the following Section 4, we obtain the conditions where the manufacturer can benefit from providing consumer returns policy. In Section 5, we give numerical examples to illustrate how consumer return rate impact the equilibrium results. The last section summarizes the research findings and future research directions.
2. Problem statement and model

There are two competing supply chains in the same market, where the manufacturer sales the substitutable product directly to customers in one single sale period, and accept the customer returns. Such policy can improve manufacturer’s profit through stimulating demand, but it can also increase the manufacturer’s processing cost because of the higher customer returns. The problem defined in this paper is that how to take the effective measure to balance the positive effect and the negative effect of customer returns.

In the market, customers who choose the product in which one of supply chains consider on the retail price $p_i$ ($i=1, 2$) and return price $r_i$ ($i=1, 2$), which is refund amount the manufacturers give back to customers. Let $r_i \leq p_i$ ($i=1, 2$).

We assume that the retailer’s demand $D_i(p, r)$ ($i=1, 2$) is related with both retail price and return price in two supply chains, and it is the linear function of them. That is

$$D_i(p, r) = a - bp_i + cp_j + \xi r_i - \eta r_j, \quad j = 3 - i. \quad (1)$$

Because the retail price limits the range of the return price, we can assume $r_1 = \beta_1 p_1, r_2 = \beta_2 p_2$, where $0 \leq \beta_1, \beta_2 \leq 1$. $\beta_1 = \beta_2 = 0$ denotes neither of supply chains chooses customer returns policy. $\beta_1 = \beta_2 = 1$ denotes both supply chains choose the full refund policy. Generally, $0 < \beta_1, \beta_2 < 1$ denotes that both supply chains choose the partial refund policy. So, the $i$th chain’s demand function becomes

$$D_i(p, r) = a - (b - \xi \beta_i) p_i + (c - \eta \beta_j) p_j. \quad (2)$$

Let the parameters $a, b > 0, c \geq 0, \xi \geq 0, \eta \geq 0$. $a$ is the initial market demand, which reflects the whole developing level of the products. $b$ denotes the demand responsiveness to the supply chain’s retail price, while $c$ denotes the demand responsiveness to its rival’s retail price. $\xi$ denotes the demand responsiveness to the supply chain’s return price, while $\eta$ denotes the demand responsiveness to its competitor’s return price. The parameters $\beta_1, \beta_2$ are called return factors, and reflect the size of return price. We require $\xi < b, \eta < c, b - \xi \beta_1 > c - \eta \beta_2$ and $b - \xi \beta_2 > c - \eta \beta_1$.

In this paper, the production cost is not considered and products have no salvage value at the end of sales period. We assume the inventory quantity is adequate, and the return rate is $H_i, i=1, 2$, respectively.
Then, the $i$th chain’s profit is

$$\Pi_i = (p_i - \beta_i p_i H_i) D_i(p) = (1 - \beta_i H_i) p_i [a - (b - \xi \beta_i) p_i + (c - \eta \beta_i) p_j].$$  

(3)

3. Supply chain competition equilibrium results

In this section, we analyze equilibrium results in Bertrand-Nash game and Stackelberg game respectively.

3.1. The equilibrium results in Bertrand-Nash game

In Bertrand-Nash game, each supply chain decides its retail price conditional on that of its competing supply chain, which develops a duopoly market. Therefore, the $i$th chain’s equilibrium retail price

$$p_{ib} = [(c - \eta \beta_i) + 2(b - \xi \beta_i)] U a.$$  

(4)

Where $U = [4(b - \xi \beta_2)(b - \xi \beta_2) - (c - \eta \beta_1)(c - \eta \beta_2)]^{-1}$.

The $i$th chain’s equilibrium demand becomes

$$D_{ib} = (b - \xi \beta_i)[2(b - \xi \beta_i) + (c - \eta \beta_i)] U a.$$  

(5)

The $i$th chain’s equilibrium profit is

$$\Pi_{ib} = (1 - \beta_i H_i) a^2 U^2 (b - \xi \beta_i)[2(b - \xi \beta_i) + (c - \eta \beta_i)]^2.$$  

(6)

3.2. The equilibrium results in Stackelberg game

In Stackelberg game, without loss of generality, we assume that supply chain 1 is a leader and supply chain 2 is a follower. Then, the decision sequence is that supply chain 1 firstly set the retail price $p_1$, and then supply chain 2 chooses the optimal retail price to make its profit $\Pi_2 (p_1, p_2)$ maximize. Then, the $i$th chain’s optimal retail price is

$$p_{is} = a [(b - \xi \beta_i) + (c - \eta \beta_i)] U_0.$$  

(7)

The $i$th chain’s equilibrium demand is

$$D_{is} = U_0 a (b - \xi \beta_1)(b - \xi \beta_2).$$  

(8)
Where $U_0 = \left[ 2(b - \xi \beta_1)(b - \xi \beta_2) - (c - \eta \beta_1)(c - \eta \beta_2) \right]^{-1}$. Therefore, in Stackelberg game, the $i$th chain’s equilibrium profit is

$$
\Pi_{iS} = (1 - \beta_i H_i) U_0^2 a^2 \left[ (b - \xi \beta_j) + (c - \eta \beta_j) \right] (b - \xi \beta_j)(b - \xi \beta_j).
$$

(9)

### 3.3. Analyze the impact of return price on retail price, order quantity and profits

In this subsection, we analyze the impact of return price on the optimal retail price, order quantity and profits, and we also compare the optimal price, order quantity and profits in Stackelberg game with that in Bertrand-Nash game. We obtain some conclusions as follows.

**Proposition 1**

In Bertrand-Nash game and Stackelberg game, the optimal retail price $p_{1B}$, $p_{1S}$ increase in $\beta_1$, and the optimal retail price $p_{2B}$, $p_{2S}$ increase in $\beta_2$. If $c \xi > b \eta$, then the optimal retail price $p_{1B}$, $p_{1S}$ increase in $\beta_2$, and the optimal retail price $p_{2B}$, $p_{2S}$ increase in $\beta_1$. Otherwise, if $c \xi < b \eta$, then the optimal retail price $p_{1B}$, $p_{1S}$ decrease in $\beta_2$, and the optimal retail price $p_{2B}$, $p_{2S}$ decrease in $\beta_1$.

From Proposition 1, in Bertrand-Nash game and Stackelberg game, we conclude that the optimal retail price of each supply chain increase in its own return price. This suggests that two competing supply chains should improve their retail price to win the profit, when they increase the return price. The change of the optimal retail price with the return price of its competing one is dependent on $c \xi$ and $b \eta$.

**Proposition 2**

$p_{1S} > p_{1B}$, $p_{2S} > p_{2B}$. Furthermore, if $\beta_2 < \beta_1$, then $p_{2S} < p_{1S}$ and $p_{1B} < p_{2B}$. Otherwise, if $\beta_2 > \beta_1$, then $p_{2S} > p_{1S}$ and $p_{2B} > p_{1B}$.

Proposition 2 shows that in Stackelberg game, the retail price in two supply chains is higher than corresponding that in Bertrand-Nash game. In addition, both in Stackelberg game and Bertrand-Nash game, if the return price in supply chain 1 is higher (lower) than that in supply chain 2, then the corresponding retail price is also higher (lower).
Proposition 3

If \( c \xi > b \eta \), then the optimal order quantity \( D_{1B}, D_{2B}, D_{1S} \) and \( D_{2S} \) increase in \( \beta_1 \) and \( \beta_2 \). Otherwise, if \( c \xi < b \eta \), then the optimal order quantity \( D_{1B}, D_{2B}, D_{1S} \) and \( D_{2S} \) decrease in \( \beta_1 \) and \( \beta_2 \).

Proposition 4

\( D_{1S} < D_{1B}, D_{2S} < D_{2B}, D_{1S} = D_{2S} \). Furthermore, if \( \beta_2 < \beta_1 \), then \( D_{1B} < D_{2B} \); Otherwise, if \( \beta_2 > \beta_1 \), then \( D_{1B} > D_{2B} \).

From Proposition 4, we know that overall in Bertrand-Nash game the order quantity in two supply chains is higher than corresponding that in Stackelberg game. In additions, in Stackelberg game, the order quantities in two competing supply chains are same. In Bertrand-Nash game, the changing trends of the optimal order quantity are opposite with that of the return price.

Proposition 5

If \( c \xi > b \eta \), then the optimal profit \( \Pi_{1B}, \Pi_{1S} \) increase in \( \beta_2 \), and the optimal profit \( \Pi_{2B}, \Pi_{2S} \) increase in \( \beta_1 \). Otherwise, if \( c \xi < b \eta \), then the optimal profit \( \Pi_{1B}, \Pi_{1S} \) decrease in \( \beta_2 \), the optimal profit \( \Pi_{2B}, \Pi_{2S} \) decrease in \( \beta_1 \).

Proposition 6

\( \Pi_{1S} > \Pi_{1B}, \Pi_{2S} > \Pi_{2B} \). Furthermore, in the case of \( c \xi > b \eta \), if \( \beta_2 < \beta_1 \), then \( \Pi_{2S} > \Pi_{1S} \) and \( \Pi_{2B} > \Pi_{1B} \). Otherwise, if \( \beta_2 > \beta_1 \), then \( \Pi_{2S} < \Pi_{1S} \) and \( \Pi_{2B} < \Pi_{1B} \). In the case of \( c \xi < b \eta \), we have \( \Pi_{2S} < \Pi_{1S} \) and \( \Pi_{2B} < \Pi_{1B} \).

Proposition 6 suggests that the profits of two competing supply chains in Stackelberg game are larger than corresponding that in Bertrand-Nash game. Furthermore, in the case of \( c \xi > b \eta \), when the return price of the competing supply chain is smaller (larger) than that of its own supply chain, the profits of competing supply chain are larger (smaller) than that of its own supply chain. In the case of \( c \xi < b \eta \), it has an opposite trend.
4. Managerial Insights

From the above Section, we can see the consumer return rate $H$ impacts on the manufacturer’s profit. Therefore, in this section, let $\beta_1=\beta_2=0$ and we obtain the equilibrium profits without the manufacturers’ consumer returns policy in Bertrand-Nash game and Stackelberg game, respectively. Then, we will further to investigate in what cases the manufacturer can be beneficial for providing the customer returns policy.

Without consumer returns policy, the manufacturers’ equilibrium profits in Bertrand-Nash game are same, and they are

$$\Pi_{0B} = \frac{a^2b}{(2b-c)^2}.$$ 

The manufacturers’ equilibrium profits in Stackelberg game are same, and they are

$$\Pi_{0S} = \frac{a^2b^2(b+c)}{(2b^2-c^2)^2}.$$ 

Comparing $\Pi_{iB} (i=1,2)$ with $\Pi_{0B}$, we obtain Proposition 7 as follows.

**Proposition 7**

In Bertrand-Nash game, the manufacturers can make more profits from consumer returns policy than that without it, only if

$$0 < H_i < \frac{1}{\beta_i} \left\lfloor 1 - \frac{bU^{-2}}{(b-\xi\beta_i)(2b-c)^2[2(b-\xi\beta_i)+(c-\eta\beta_i)]^2} \right\rfloor.$$ 

Proposition 7 shows that in Bertrand-Nash game, if consumer return rate is sufficiently low such that $H_i < \frac{1}{\beta_i} \left\lfloor 1 - \frac{bU^{-2}}{(b-\xi\beta_i)(2b-c)^2[2(b-\xi\beta_i)+(c-\eta\beta_i)]^2} \right\rfloor$, the manufacturer can be beneficial from consumer returns policy. Otherwise, if consumer return rate is sufficiently high such that $H_i > \frac{1}{\beta_i} \left\lfloor 1 - \frac{bU^{-2}}{(b-\xi\beta_i)(2b-c)^2[2(b-\xi\beta_i)+(c-\eta\beta_i)]^2} \right\rfloor$, consumer returns policy cannot make the manufacturers benefit, and the manufacturers should take no returns policy.

Comparing $\Pi_{iS} (i=1,2)$ with $\Pi_{0S}$, we obtain Proposition 8 as follows.
Proposition 8

In Stackelberg game, the manufacturers can make more profits from consumer returns policy than that without it, only if

\[
0 < H < \frac{1}{\beta} \left[ 1 - \frac{U_{0}^{2}b^{2}(b+c)}{(2b^{2}-c^{2})^{2}} \left( (b-\xi\beta_{j})+(c-\eta\beta_{j}) \right)(b-\xi\beta_{j})(b-\xi\beta_{j}) \right] \].

Proposition 8 shows the similar conclusion with Proposition 7. That is, in Stackelberg game, the manufacturers can be beneficial from consumer returns policy, only if consumer return rate is sufficiently low.

5. Numerical example

In this section, through numerical examples, we will illustrate how the return price impact on the supply chain’s optimal retail price, order quantity and profit. Then we also investigate how the change of return rate impacts the optimal profit.

5.1 The impact of the return price on the optimal retail price

In the basic model, we let parameters \( a = 1000 \), \( b = 20 \), \( c = 15 \), \( \xi = 8 \) and \( \eta = 5 \). These can make sure two supply chains positive profit. When \( \beta_{1} = 0.8 \) and \( \beta_{2} \) changes from 0 to 1, we analyze how the retail price changes with the return price.

![Figure 1. The change of the retail price with the return price in two games](image)

From Figure 1, we can obtain \( \frac{\partial p_{2s}}{\partial \beta_{2}} > \frac{\partial p_{2b}}{\partial \beta_{2}} \) and \( \frac{\partial p_{1s}}{\partial \beta_{2}} > \frac{\partial p_{1b}}{\partial \beta_{2}} \). It shows that in Stackelberg game the retail price is more seriously impacted by the return price than that in Bertrand game, whether from its own chain or from the competing chain.
5.2 The impact of the return price on the optimal order quantity

![Figure 2. The change of the order quantity with the return price in two games](image)

From Figure 2, we can obtain that $\frac{\partial D_{1b}}{\partial \beta_2} > \frac{\partial D_{1s}}{\partial \beta_2}$ and $\frac{\partial D_{2s}}{\partial \beta_2} > \frac{\partial D_{2b}}{\partial \beta_2}$. It shows that in Stackelberg game the retail price is impacted by its own chain’s return price is larger than that in Bertrand-Nash game. However, it is impacted by the return price of its competing chain in Stackelberg game is smaller than that in Bertrand-Nash game.

5.3 The impact of the return price on the optimal profit

In the basic model, we let parameters $a = 1000$, $b = 20$, $c = 15$, $\xi = 8$ and $6$, $\eta = 5$, $H_1 = H_2 = 0.9$ and $0.1$. When both $\beta_1$ and $\beta_2$ change from $0$ to $1$, we will analyze the change of $\Pi_{1B}$, $\Pi_{2B}$, $\Pi_{1s}$ and $\Pi_{2s}$ in four cases.

In the model of this paper, the structure of two competing supply chain is symmetric whether in Stackelberg game or in Bertrand-Nash game. Therefore, we only analyze the change of the profits in one of supply chain, and we can obtain the change of the profits in the other chain through exchanging $\beta_1$ with $\beta_2$.

**Case 1.** When $c\xi > b\eta$ ($\xi = 8$) and $H_1 = H_2 = 0.1$, we can have Figure 3 as follows.

![Figure 3. The change of the optimal profits with the return price in two games](image)
From Figure 3, we know that when \( c \xi > b \eta \) and the return rate is lower, the maximum profits of two supply chains can be achieved at \( \beta_1 = \beta_2 = 1 \).

**Case 2.** When \( c \xi > b \eta (\xi = 8) \) and \( H_1 = H_2 = 0.9 \), we can obtain Figure 4.

![Figure 4](image1)

*Figure 4. The change of the optimal profits with the return price in two games*

From Figure 4, we know that when \( c \xi > b \eta \) and the return rate is higher, the maximum profits of two supply chains can be achieved where the own supply chain takes no return policy and the competing supply chain takes the full refund policy.

**Case 3.** When \( c \xi < b \eta (\xi = 6) \) and \( H_1 = H_2 = 0.1 \), we can have Figure 5.

![Figure 5](image2)

*Figure 5. The change of the optimal profits with the return price in two games*

From Figure 5, we know that when \( c \xi < b \eta \) and the return rate is lower, the maximum profits of two supply chains can be achieved where the own supply chain takes full refund policy and the competing supply chain takes no return policy.
Case 4. When $c_{\xi} < b\eta \left(\xi = 6\right)$ and $H_1 = H_2 = 0.9$, we can obtain Figure 6.

![Figure 6](image)

Figure 6. The change of the optimal profits with the return price in two games

From Figure 6, we know that when $c_{\xi} < b\eta$ and the return rate is higher, the maximum profits of two supply chains can be achieved at both the competing supply chains taking no return policy.

From the Figures, besides the conclusion of Proposition 5 and Proposition 6, we can also know that
\[
\frac{\partial \pi_{1s}}{\partial \beta_2} > \frac{\partial \pi_{1b}}{\partial \beta_2}, \frac{\partial \pi_{2s}}{\partial \beta_2} > \frac{\partial \pi_{2b}}{\partial \beta_2}.
\]
These suggest that the profits of two competing supply chains in Stackelberg game are affected greater by the return price than that in Bertrand-Nash game.

5.4 The impact of the customer return rate on the optimal profit

Because of the symmetry of supply chains, we can need to illustrate the impact of the return rate $H_2$ on the optimal profit $\Pi_{2B}$ and $\Pi_{2S}$. Here, we obtain Figure 7.

![Figure 7](image)

Figure 7. The change of the optimal profits with the return rate in two games

Figure 7 shows that when the customer return rate changes from 0 to 1, how the profits of supply chains change with their return price both in Stackelberg game and in Bertrand-Nash game. From the above Figure, we can see when customer return rate is lower, the optimal profit increases with the return price both in Stackelberg game and in Bertrand-Nash game.
However, when customer return rate is higher, the optimal profit decreases with the return price. Furthermore, the results above suggest that the profits of two competing supply chains are influenced simultaneously by the customer return rate and the return price of supply chain.

6. Conclusion

Customer returns policy is an important decision for firms to strive and develop in the fiercely competitive market. In this paper, we first solve the equilibrium results with the manufacturers’ consumer returns policy in Bertrand-Nash game and Stackelberg game. Then comparing them with that without consumer returns policy, we find that the manufacturers can make benefits from consumer returns policy, only if consumer return rate is sufficiently low. In additions, we analyze the impact of the return price on the optimal retail price, the optimal order quantity and the optimal profit. Through the propositions and the numerical analysis, we conclude that the retail price and order quantity are affected only by return price, but the profits of supply chains are influenced by both return price and customer return rate. In addition, retail price, order quantity and profits of two competing supply chains in Stackelberg game are affected greater by the change of return price than that in Bertrand-Nash game.

In the future work, we will further consider when the demand is dependent on the return price and retail price, but the return price is independent with the retail price, the optimal pricing and ordering decision. In additions, incorporating the strategic consumer behavior into the consumer returns model in a duopoly setting is a future research direction.

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