

Optimal manufacturing and remanufacturing capacities of systems with reverse logistics and deterministic uniform demand

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Abstract: Using reverse logistics in production systems can help to reduce costs. However, it can also mean introducing a source of uncertainty in the system behavior. In this study we present a method for calculating the optimal manufacturing and remanufacturing capacities of a system with reverse logistics and steady demand taking into account the random behavior of the quantity, quality and timing of units that are collected thru the reverse logistics system. The collected units are remanufactured or disposed of. We also provide an example to illustrate the method.

Keywords: reverse logistics, remanufacturing, capacity planning

1 Introduction

When the reverse logistics is implemented in a production system, managers must take into account, in their decisions, that the operations of the system have been affected. If they want to operate optimally, the structure of the new system must be adapted to reflect the new situation. In particular, the manufacturing capacity should be modified as the reverse logistics gives a new source of supply: the remanufactured units that are collected at the end of the useful life of a product.

The recovery of the waste produced at the end of a product life cycle has become a common task in industry. Companies must comply with environmental regulations, according to which the manufacturer of a product is responsible for managing the waste that it generates. However, it is now possible to make a profit from managing products outside their useful life cycle if the process is planned appropriately (Rubio, 2003).

Reverse logistics seek to respond to new logistics management requirements for processing products at the end of their useful lives. According to de Britto and Dekker (2004), the European Working Group on Reverse Logistics (REVLOG) defined the reverse logistics as:

“The process of planning, implementing and controlling backwards flows of raw materials, in process inventory, packaging and finished goods, from a manufacturing, distribution or use point, to a point of recovery or point of proper disposal”

Thierry et al. (1995) introduce the concept of “product recovery management” and lists five product recovery options: repair, refurbishing, remanufacturing cannibalization and recycling.

According to Guide (2000), a recoverable manufacturing system is characterized by the uncertainty in timing, quantity and quality of returns and the place where to collect them. This uncertainty affects planning performance and production control (Prahinski and Kocabasoglu, 2006) and creates unpredictability in remanufactured product inventories.

Rubio et al. (2008) analyze the main characteristics of articles on reverse logistics and Fleischmann and Minner (2003) classify the models for describing systems with reverse logistics in two categories: deterministic models (for example Minner and Kleber (2001) and Choi *et al.* (2007)) and stochastic models (for example Fleischmann and Kuik, 2003, van der Laan *et al.*, 2004, Buchanan and Abad, 1998 and Teunter, 2006). These models are used to determine optimal ordering and inventory policies supposing that manufacturing and remanufacturing capacities are enough to supply the demand. Vlachos *et al.* (2007), Georgiadis *et al.* (2006) and Georgiadis and Vlachos (2004) present dynamic models for strategic remanufacturing and collection capacity planning where these capacities are state

variables and a policy is proposed in order to calculate them. Another relevant reference is Souza (2008), which mentions other sources regarding single period models and used product acquisition.

In this paper we study a production system with constant demand and stochastic returns for a single product and we focus in determining the optimal manufacturing and remanufacturing capacities. We also analyze the effects of random remanufacturing factors on the system performance.

In Section 2 we describe the system and outline the conditions of the parameters involved, considering two different scenarios. In the first one the company meets all demand and in the second scenario not all demand is necessarily met. In section 3 we describe the manufacturing and remanufacturing policy for the first scenario, provide an approximation of the probability distribution used to determine the amount and rate of collected products, present an algorithm for calculating the optimal manufacturing and remanufacturing capacities; and we calculate optimal values for a specific case study. In section 4 we give an iterative process to determine the manufacturing and remanufacturing capacities for the second scenario. In Section 5 we describe how to determine the optimal manufacturing and remanufacturing capacities when there are n different quality types of collected products. Finally, in Section 6 we present the main conclusions of the study.

2 System description

We consider a system that produces and sells a single product. The product can be returned to the company once it has completed its useful life. The collected units are remanufactured and resold as new or are disposed of. The system has the following features:

- The time horizon of the system is discrete with periods of equal length.
- The company makes the decisions at the end of each period.
- The demand D (units/period) is known and is the same in each period.
- It is a just-in-time production system, so there should be no inventories.

- The system has unknown and limited manufacturing and remanufacturing capacities, namely X and Y units per period respectively which will be calculated by minimizing costs. It is assumed that:
 - X and Y do not change over time.
 - There is sufficient capacity to supply the demand, i.e. $X + Y \geq D$.
 - $X \leq D$ and $Y \leq D$ because never will be used capacities greater than D in order to supply the demand.

The production costs of the original production system and the remanufacturing system are composed of fixed costs C_p and C_r (which depend on the installed capacity and, therefore, do not vary provided that the production capacity remains constant) and variable costs (per unit of output) c_p and c_r . It is assumed that C_p is an increasing function of X and C_r is an increasing function of Y .

The returns have the following characteristics:

- The end-of-usage of the product occurs between periods T_1 and T_2 after it is sold. p_i is the probability that the end-of-usage of the product occurs i periods after it is sold ($i = T_1, \dots, T_2$).
- r is the probability of an end-of-usage product being returned and collected. Therefore, the probability that a unit sold in period t will be collected in period $t+i$ is $r \cdot p_i$.
- There is only one quality type for collected products. Therefore, each unit of collected product undergoes the same remanufacturing process.
- The collected units are remanufactured or are disposed of. A remanufactured product has the same usage expectancy and return quality as a manufactured product.
- Each collected unit has a cost of c_{rc} .
- The cost of disposing of collected units is zero.

If we assume that there is no product recovery, the optimal inventory policy is such that the inventory costs are zero. Therefore, the costs for each period would be $C_p(D) + c_p \cdot D$. When products are collected and remanufactured, the company can sell units from either the original production system or the remanufacturing system.

Since there is an inherent degree of uncertainty in the availability of returns, we analyze two different scenarios. In the first one there is a supplier with sufficient capacity that enables the company meet all demand with a cost per unit of c_s ; it is assumed that c_s is greater than c_p and c_r . In the second scenario the company's production policy can sometimes cause supply interruptions; in this case, the unmet demand is lost at a shortage cost per unit, b ; it is assumed that b is greater than c_p and c_r .

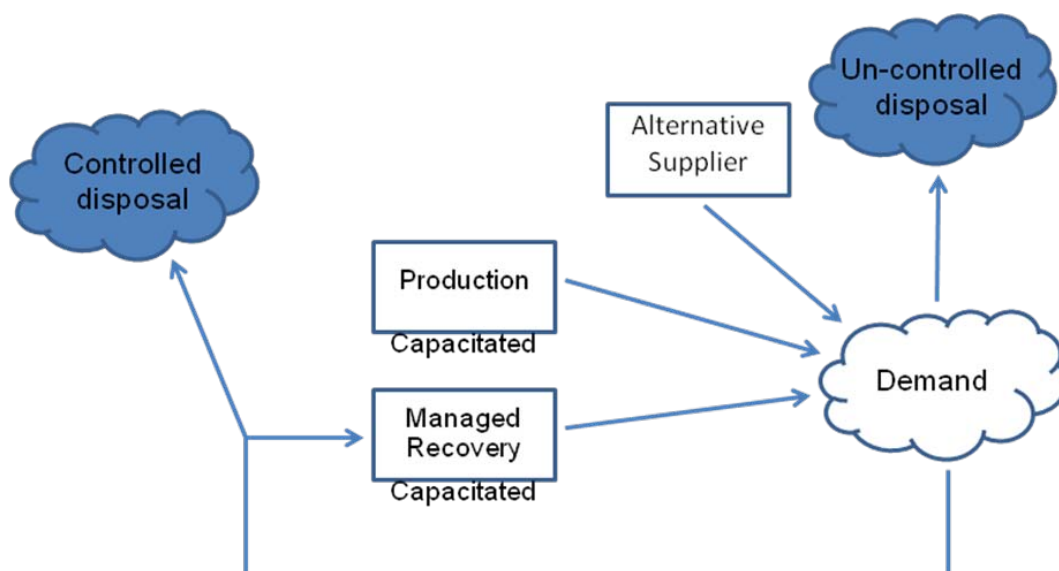


Figure 1. "Schematic representation of the system". Source: Own contribution.

3 Determining the optimal manufacturing and remanufacturing capacities of a system with alternative supplier

The optimal manufacturing and remanufacturing capacities are calculated by minimizing the expected value of the cost incurred in each period according to the following process: first, for each pair of manufacturing and remanufacturing capacities we calculate the minimum expected cost incurred by a system with these capacities (to do this we must determine, for each pair of capacity values,

the optimal manufacturing and remanufacturing policy and calculate the expected value of the cost associated with the optimal policy); next, in order to determine the optimal capacities, we choose the capacities associated with the minimum of the expected costs calculated in the previous step.

The costs incurred by the company during each period depend on the quantity of goods manufactured, collected and remanufactured by the company and on the goods purchased from the supplier. These amounts will be limited by the installed manufacturing and remanufacturing capacities and by the quantity of collected product, which is a random value.

3.1 Optimal manufacturing and remanufacturing policy

The manufacturing and remanufacturing policy is obtained by solving the mathematical program shown below, given the manufacturing and remanufacturing capacities X and Y , and the units of product collected during each period, d :

$$[\text{MIN}] c = C_p(X) + C_r(Y) + c_p \cdot X + c_r \cdot y + c_s \cdot (D - x - y) + c_{rc} \cdot d$$

s.t.:

$$\begin{aligned} x + y &\leq D \\ x &\leq X \\ y &\leq \min\{Y, d\} \\ x, y &\geq 0 \end{aligned}$$

Where x and y are the quantities of product to manufacture and remanufacture respectively. The optimal solution depends on the values of d , X , Y and D , and also on the relation between c_r and c_p .

When $c_r < c_p$, three cases can be distinguished:

1. $d < D - X$: The company meets total demand using the alternative supplier. The optimal values and costs incurred are:

$$x = X, \quad y = d$$

$$c = C_p(X) + C_r(Y) + (c_p - c_s) \cdot X + c_s \cdot D + (c_r - c_s + c_{rc}) \cdot d$$

2. $D - X \leq d < Y$: The optimal values and costs incurred are:

$$x = D - d, \quad y = d$$

$$c = C_p(X) + C_r(Y) + c_p \cdot D + (c_r - c_p + c_{rc}) \cdot d$$

3. $d \geq Y$: The returns are higher than Y . The optimal values are:

$$x = D - Y, \quad y = Y$$

$$c = C_p(X) + C_r(Y) + c_p \cdot D + (c_r - c_p) \cdot Y + c_{rc} \cdot d$$

When $c_r \geq c_p$ we have two cases:

1. $d < D - X$. The optimal values and costs incurred are:

$$x = X, \quad y = d$$

$$c = C_p(X) + C_r(Y) + (c_p - c_s) \cdot X + c_s \cdot D + (c_r - c_s + c_{rc}) \cdot d$$

2. $D - X \leq d$. In this case, the optimal values and costs incurred are:

$$x = X, \quad y = D - X$$

$$c = C_p(X) + C_r(Y) + (c_p - c_r) \cdot X + c_r \cdot D + c_{rc} \cdot d$$

3.2 Probability distribution of collected product quantity

The quantity of product collected during a given period from the quantity of product sold in the i -th previous period follows a binomial distribution $B(D, r \cdot p_i)$, where p_i is the probability that the product will come to the end of its useful life during the i -th period after its sale; r is the probability that the product will be returned once it has completed its useful life; and D is the quantity of product sold during the i -th previous period.

The quantity of product collected during a given period is equal to the sum of the collected products from each of the previous periods. The probability that this value will be d is denoted by $p(d)$.

The expected value of combined manufactured and remanufactured products from the company is called PM and is calculated using the following expression:

$$PM = D - \sum_{d=0}^{D-X} (D - X - d) p(d)$$

When $r \cdot p_i$ is sufficiently small, we can approximate the probability distribution of collects from a given period to a Poisson distribution with parameter $D \cdot r \cdot p_i$. Therefore, the total amount of product units collected during a given period follows a Poisson distribution with parameter $r \cdot D$ (since the sum of p_i is 1). In this case we obtain:

$$p(d) = \frac{e^{-rD} (rD)^d}{d!}$$

3.3 Calculating the optimal manufacturing and remanufacturing capacities

If we assume the manufacturing and remanufacturing policy established in Section 3.1 and the probability distribution of collected product defined in Section 3.2, we can determine the expected value of the cost function by using the following expression:

$$E(c(d)) = \sum_{d=0}^{\infty} c(d) \cdot p(d)$$

Case $c_r < c_p$:

$$E(c(d)) = C_p(X) + C_r(Y) + \sum_{d=0}^{D-X} [(c_p - c_s) \cdot X + c_s \cdot D + (c_r - c_s + c_{rc}) \cdot d] \cdot p(d) + \sum_{d=D-X}^Y [c_p \cdot D + (c_r - c_p + c_{rc}) \cdot d] \cdot p(d) + \sum_{d=Y}^{\infty} [c_p \cdot D + (c_r - c_p) \cdot Y + c_{rc} \cdot d] \cdot p(d)$$

By reordering the terms we obtain:

$$E(c(d)) = c_p \cdot D + c_{rc} \cdot E(d) + C_p(X) + (c_s - c_p) \sum_{d=0}^{D-X} (D - X - d) \cdot p(d) + C_r(Y) - (c_p - c_r) \cdot \left[Y - \sum_{d=0}^Y (Y - d) \cdot p(d) \right]$$

Where $E(d)$, the expected value of d , is equal to $r \cdot D$. We can then define the following functions for determining the optimal solution:

$$g_1(X) = C_p(X) + (c_s - c_p) \sum_{d=0}^{D-X} (D - X - d) p(d)$$

$$g_2(Y) = C_r(Y) - (c_p - c_r) \cdot \left[Y - \sum_{d=0}^Y (Y - d) p(d) \right]$$

$$g(X, Y) = c_p \cdot D + c_{rc} \cdot r \cdot D + g_1(X) + g_2(Y)$$

Therefore, the desired values of X and Y are the solution of the following problem P:

$$[\text{MIN}] g(X, Y)$$

s.t.:

$$X \leq D$$

$$Y \leq D$$

$$X + Y \geq D$$

$$X, Y \geq 0$$

Case $c_r \geq c_p$:

The desired values of the capacities X and Y are the solution of the problem P but now with the following expressions for g_1 , g_2 and g :

$$g_1(X) = C_p(X) + (c_p - c_r) \cdot X + (c_s - c_r) \cdot \sum_{d=0}^{D-X} (D - X - d) p(d)$$

$$g_2(Y) = C_r(Y)$$

$$g(X, Y) = c_r \cdot D + c_{rc} \cdot r \cdot D + g_1(X) + g_2(Y)$$

Both cases are non-linear programming problems.

3.4 Numerical example

We analyze a company that produces and sells a product with the following features:

- Demand $D = 100$ u/period.
- Variable cost of manufacturing $c_p = €10/u$.
- Variable cost of remanufacturing $c_r = €5/u$.
- Variable collection cost $c_{rc} = €1/u$.
- Fixed manufacturing costs according to the capacity X :
 $C_p(X) = 15 \cdot X - 0,05 \cdot X^2$.
- Fixed remanufacturing costs according to the capacity Y :
 $C_r(Y) = 3 \cdot Y - 0,01 \cdot Y^2$.
- Unitary cost of supply $c_s = €30/u$.
- Probability of product returns $r = 0.3$.
- Probability distribution of collected product units: the company configuration meets the conditions for using a Poisson distribution with parameter $r \cdot D$.

The system, without including remanufacturing, will have a manufacturing capacity of 100 units with a cost of €2,000 per period. When the remanufacturing system is included, the minimum of g is reached at $(X, Y) = (72, 30)$ and its value is $g(X, Y) = €1818.70$. This gives a PM of 98.70.

Figure 1 shows the graph of the function $g(X, Y)$.

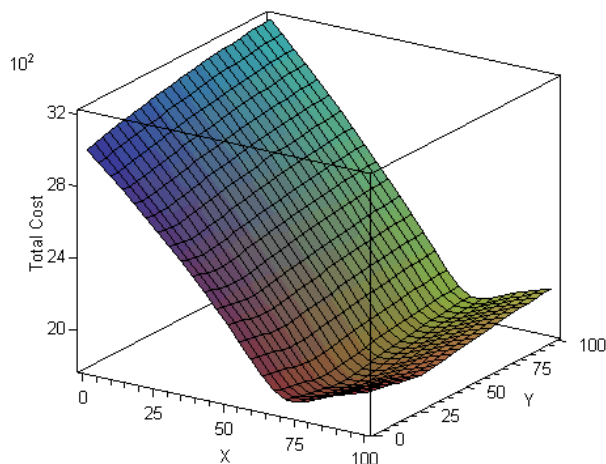


Figure 1. "Total cost depending on the manufacturing and remanufacturing capacities for $V = 98.945$ ". Source: Own contribution.

4 Determining the optimal manufacturing and remanufacturing capacities of a system without alternative supplier

The optimal manufacturing and remanufacturing policy is calculated as in section 3.1, using the same expressions and changing the unit cost of supply c_s for the unit cost of shortage b .

The quantity of product units collected during a given period from the quantity of product sold in the i -th previous period follows a binomial distribution $B(v_i, r \cdot p_i)$, where v_i is the quantity sold during the i -th previous period and r and p_i are defined as in section 3.2.

Since in this case there is a possibility of inventory shortage, the value of v_i behaves randomly and is less than D . We suppose that the system is in a stationary state and therefore the probability distributions of the sales are the same in each period. The probability distribution of the quantity of collected units, $p(d)$, depends on $q(v)$, the probability distribution of the quantity sold in any period, which, in turn, depends on $p(d)$. In order to solve this cyclic dependency we use the following iterative process (IP1) to compute $p(d)$ where $p_{in}(d_i)$ is the approximation, in the n -th iteration, of the probability of the number of collected units corresponding to the sales of the i -th preceding period is equal to d_i and $p_n(d)$ is the approximation, in the n -th iteration, of the probability of the total number of collected units is equal to d :

Step 0: Start the process with

$$q_0(v) = \begin{cases} 1 & v = D \\ 0 & v \neq D \end{cases}$$

Where $q(v)$ is the probability that the sales in a period will be v .

Step 1: Compute $p_n(d_i)$ for $i = T_1, \dots, T_2$ and $d_i = 0, \dots, D$:

$$p_{in}(d_i) = \sum_{v=k_i}^D p_{in}(d_i | v) q_{n-1}(v) = \sum_{v=d_i}^D \binom{v}{d_i} (r \cdot p_i)^{d_i} \cdot (1 - r \cdot p_i)^{v-d_i} \cdot q_{n-1}(v)$$

Compute $p_n(d)$ for $d = 0, \dots, (T_2 - T_1 + 1) \cdot D$:

$$p_n(d) = \sum_{i=T_1}^{T_2} \prod_{d_i=d} p_{in}(d_i)$$

Step 2: Calculate the PD of product sold each period $q_n(v)$ using $p_n(d)$ calculated in step 1:

$$q_n(v) = \begin{cases} 0 & v < X \\ p_n(v - X) & X \leq v < D \\ 1 - \sum_{d=0}^{D-X} p_n(d) & v = D \end{cases}$$

Step 3: Calculate the difference between $q_{n-1}(v)$ and $q_n(v)$ where difference means some measure of how much is one distribution far from the other (for example the quantity $|E(q_n(v)) - E(q_{n-1}(v))|$ can be used as a measure of the difference). If the difference is greater than a tolerance add 1 to n and go to step 1; else take as $p(d) = p_n(d)$.

The optimal manufacturing and remanufacturing capacities are calculated solving problem P of section 3.3 but replacing the unit cost of supply c_s with the unit cost of shortage b in the expression of $g_1(X)$ and replacing $c_{rc} \cdot r \cdot D$ with $c_{rc} \cdot r \cdot V$ in the expression of $g(X, Y)$, where V is the expected value of the product sold:

$$V = D - \sum_{d=0}^{D-X} (D - X - d) p(d)$$

In the case $c_r < c_p$, using the expression of V we have:

$$g_1(X) = C_p(X) + (b - c_p) \cdot (D - V)$$

$$g(X, Y) = C_p(X) + b \cdot D + (c_p + c_{rc} \cdot r - b) \cdot V + g_2(Y)$$

Analogously in the case $c_r \geq c_p$ we have:

$$g(X, Y) = C_p(X) + (c_p - c_r) \cdot X + b \cdot D + (c_r + c_{rc} \cdot r - b) \cdot V + g_2(Y)$$

When solving the problems it is important to take into account that the probability distribution of collected units $p(d)$ depends on X . So we define an iterative process (IP2) to find the optimal values:

- Step 0: Start the process with $X_0 = (1 - r) \cdot D$.
- Step 1: Compute PD of collected units using the iterative process IP1 described above.
- Step 2: Determine (X_n, Y_n) solving problem P which optimizes the value of the expected cost $g_n(X_n, Y_n)$.
- Step 3: If the desired accuracy in $g_n(X_n, Y_n)$ is not reached then go to step 1; otherwise finish the process.

We recalculate the numerical example of section 3.4 but replacing the unit cost of supply c_s with the unit cost of shortage $b = \text{€}30/\text{u}$ and with product end-of-usage occurring between periods 1 and 6 with probabilities $p_1 = 0.1$, $p_2 = 0.2$, $p_3 = 0.2$, $p_4 = 0.25$, $p_5 = 0.15$, $p_6 = 0.1$.

The minimum of g is reached at $(X, Y) = (72, 30)$ and its value is $g(X, Y) = \text{€}1820.90$. This gives a value of $V = 98.61$.

We have used the following tolerances in step 3 of each iterative process:

$$\text{For IP1: } \frac{|E(q_n(v)) - E(q_{n-1}(v))|}{E(q_{n-1}(v))} < 0.001$$

$$\text{For IP2: } \frac{|g_n(X_n, Y_n) - g_{n-1}(X_{n-1}, Y_{n-1})|}{g_{n-1}(X_{n-1}, Y_{n-1})} < 0.001$$

The main process (IP2) converges in 3 iterations and for each iteration IP1 converges in 3 iterations.

5 System with n quality types of collected products

In this section we consider a specific case in which the collected units are defined according to a series of different quality types and we calculate the optimal manufacturing and remanufacturing policy. The procedure outlined in this section can be considered a generalization of the one described in the previous section 3.1.

The configuration is similar to that of a system in which all collected products are of the same quality (Section 2). The variable remanufacturing costs are c_j , $j=1, \dots, n$, and the returns have the following characteristics:

- p_i and r are defined in the same way as for a single quality.
- There are n different quality types for collected products.
 - R_j , $j=1, \dots, n$ is the probability that a collected product is of quality type j .
 - a_j units of remanufacturing resource are required to remanufacture one unit of collected product of quality type j ($j=1, \dots, n$).

It is assumed that c_s is greater than c_p and c_j ($j=1, \dots, n$).

The manufacturing and remanufacturing policy is obtained by optimizing the linear equation shown below, given the manufacturing and remanufacturing capacities, X and Y , and the units of collected product of quality type j ($j=1, \dots, n$) available in each period, d_j :

$$[\text{MIN}] \quad c = C_p(X) + C_r(Y) + c_p \cdot x + \sum_{j=1}^n c_j \cdot y_j + c_s \cdot \left(D - x - \sum_{j=1}^n y_j \right)$$

s.t.:

$$x + \sum_{j=1}^n y_j \leq D$$

$$x \leq X$$

$$\sum_{j=1}^n a_j \cdot y_j \leq Y$$

$$y_j \leq d_j \quad j = 1, \dots, n$$

$$x, y_1, \dots, y_n \geq 0$$

Where x is the quantity of product to manufacture and y_j are the quantities of collected product of quality j ($j=1, \dots, n$) to remanufacture. By modifying slightly the notation we obtain the following formula:

$$[\text{MAX}] \quad \sum_{j=1}^{n+1} S_j \cdot y_j$$

s.t.:

$$\sum_{j=1}^{n+1} y_j \leq D$$

$$\sum_{j=1}^n a_j \cdot y_j \leq Y$$

$$y_j \leq d_j \quad j = 1, \dots, n+1$$

$$y_1, \dots, y_{n+1} \geq 0$$

Where the objective function has been reversed and the notation has been changed as follows:

- The variable x is redefined as $y_{n+1} = x$
- The objective function parameters are compacted:
 - $S_j = c_s - c_j$ for $j=1, \dots, n$
 - $S_{n+1} = c_s - c_p$

$$\circ \quad d_{n+1} = X$$

$$\circ \quad a_{n+1} = 0$$

By using the constraints of the problem, the dual problem and the complementary slackness theorem we obtain the following expressions:

$$\sum_{j=1}^{n+1} y_j \leq D$$

$$\sum_{j=1}^n a_j \cdot y_j \leq Y$$

$$y_j \leq d_j \quad j = 1, \dots, n+1$$

$$\mu_D + a_j \cdot \mu_Y + \mu_j \geq S_j \quad j = 1, \dots, n+1$$

$$(y_j - d_j) \mu_j = 0 \quad j = 1, \dots, n+1$$

$$\left(\sum_{j=1}^{n+1} y_j - D \right) \mu_D = 0$$

$$\left(\sum_{j=1}^n a_j \cdot y_j - Y \right) \mu_Y = 0$$

$$(\mu_D + a_j \cdot \mu_Y + \mu_j - S_j) y_j = 0 \quad j = 1, \dots, n+1$$

$$y_j, \mu_j, \mu_D, \mu_Y \geq 0 \quad j = 1, \dots, n+1$$

Where $\mu_j, \mu_D, \mu_Y \geq 0 \quad j = 1, \dots, n+1$ are the dual variables. Four different cases can be distinguished depending on the values of d_j ($j=1, \dots, n+1$), Y and D :

1. The company is unable to cover all demand and all collected products can be remanufactured. Then,

$$\sum_{j=1}^{n+1} d_j < D \quad \text{and} \quad \sum_{j=1}^n a_j \cdot d_j < Y$$

And the optimal values are:

- $\mu_D = \mu_Y = 0$
- $y_j = d_j, \mu_j = S_j, j = 1, \dots, n+1$

2. The company is unable to cover all demand and not all collected products can be remanufactured. Then,

$$\sum_{j=1}^{n+1} d_j < D \text{ and } \sum_{j=1}^n a_j \cdot d_j \geq Y$$

The optimal values are:

$$y_{n+1} = d_{n+1}, \mu_{n+1} = S_{n+1}$$

Defining:

$$\alpha_j = S_j / a_j$$

There is a subscript k such that the optimal solution is:

- $\mu_D = 0$
- $y_j = d_j, \mu_j = a_j (\alpha_j - \alpha_k)$ if $\alpha_j > \alpha_k$
- $y_k = \frac{1}{a_k} \left(Y - \sum_{j=1}^{k-1} a_j \cdot d_j \right) \leq d_k, \mu_k = 0$
- $y_j = 0, \mu_j = 0$ if $\alpha_j \leq \alpha_k$

3. The company can cover all demand and all collected products can be remanufactured. Then

$$\sum_{j=1}^{n+1} d_j \geq D \text{ and } \sum_{j=1}^n a_j \cdot d_j < Y$$

Optimal values: there is a subscript k such that the optimal solution is:

$$\mu_D = S_k, \mu_Y = 0$$

- $y_j = d_j$, $\mu_j = S_j - S_k$ if $S_j > S_k$
- $y_k = D - \sum_{j=1}^{k-1} d_j \leq S_k$, $\mu_k = 0$
- $y_j = 0$, $\mu_j = 0$ if $S_j \leq S_k$

4. The company can cover all demand but not all collected products can be remanufactured. Then

$$\sum_{j=1}^{n+1} d_j \geq D \text{ and } \sum_{j=1}^n a_j \cdot d_j \geq Y$$

Optimal values: no analytical expression could be found for optimal solution and must be calculated case by case.

6 Conclusions

In the preceding sections we studied the behavior of a system with reverse logistics for manufacturing and remanufacturing a product under steady demand. The optimal manufacturing policy is constant when there is no reverse logistics, the company satisfies all the demand and no inventories are required.

We can draw several conclusions about the effects of uncertainty on the amount and rate of returns in the system and use them to compare it with an equivalent system without reverse logistics. First of all we saw that the optimal manufacturing policy becomes more complex when the system has to take into account product returns. Also, using the method that has been described for calculating the optimal manufacturing and remanufacturing capacities, we found that the manufacturing capacity can be set at a lower value than the demand and so the demand could not be totally met unless we use an alternative supplier. Finally, if the company could operate with inventories, the optimal capacities could change, so the uncertainty on returns also influences the inventory system.

The basic structure of the problem P, in section 3.3 is closely related to the newsvendor problem and its applications in stochastic inventory control. We would expect it to be possible to find structural properties of solutions to this problem

which considerably would help to solve it. We leave this study as a topic of future research.

In the last section, we described a system with n different return qualities and determined the optimal policy for a given period. We saw that the complexity increases and that could be optimal to remanufacture although the cost of remanufacture were higher than the original manufacturing costs.

A continuation of this work is study the dependency of the optimal manufacturing and remanufacturing capacities with parameters r and c_{rc} . Another extension of this research is the calculation of the optimal manufacturing and remanufacturing capacities in a system with stochastic demand.

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