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Production Sequence Determination to Minimize the Required Storage Space for the Multiple Items Production System

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Abstract:

Purpose: The research studies the production system having multiple items being processed on the same production line. The objectives are to (1) investigate the influence of production sequence on the optimal value of production run size, (2) explore the effect of production sequence on the maximum inventory level, which can affect the storage space required, and (3) propose a method to determine the proper production sequence in order to minimize the required storage space.

Design/methodology/approach: Finding that the optimal production sequence, which yields the lowest storage space required, is independent of the production run size, the research problem is divided into two independent subproblems. The first subproblem is to determine the optimal production run size to minimize the total variable cost. Here, the solution obtained from the classical multiple items EPQ model still holds. The second subproblem is to explore the proper production sequence in order to minimize the storage space required. The relationship between the production sequence and the value of maximum inventory level is determined and formulated. To explore the proper production sequence, a genetic algorithm is developed. For the performance evaluation, two experimental studies are conducted. The first experiment is to compare the solution obtained from the proposed method with the optimal solution yielded from the enumeration method on 360 small size problems. The second experiment is conducted on 180 large size problems. The result obtained from the proposed method is compared with the result yielded from the Largest Pi First (LPF) heuristic constructed by arranging the production of each item according to its production rate in non-increasing order.

Findings: It has been found that the optimal production sequence is independent of the production run size. Nonetheless, different production sequences yield different required storage spaces. With the proper production sequence, the manufacturer can reduce the total space required to keep its inventory. The proposed genetic algorithm can be applied to determine the proper production sequence in a reasonable amount of time. For the small size problem of 8 and 10 production items, the 95% confidence interval on mean of the percentage deviation between the solution yielded from the proposed genetic algorithm and the optimal solution is (0.0015, 0.0123). For the large size problem of 15 production items, the proposed genetic algorithm provides the better solution than the LPF heuristic for 158 out of 180 problems with the 95% confidence interval on mean of the percentage deviation of (5.5629, 7.0435). For those remaining 22 problems, the two methods yield the same results. In comparison to the LPF heuristic, the benefit of genetic algorithm is more pronounced when the slack proportion is getting smaller.

Research limitations/implications: According to the research model, no shortages are allowed. Therefore, the model is applicable for the production system having the summation value of the ratio between demand rate and production rate for all items not greater than one.

Originality/value: Those traditional research involving the determination of optimal production run size and production sequence in the system having multiple items being produced on the same production line differs from each other in their production environments. However, most of them still have the objective function of minimizing the total system cost incurred. To the best of our literature searching, none of them discussed the influence of production sequence on the total inventory level, which directly affects the required storage space, one of the critical issues facing by many manufacturers. The originality of this work is to show that different production sequence yields different total storage space required and proposed the method to determine proper production sequence.

Keywords: inventory management, production sequencing, genetic algorithm, multiple items

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1. Introduction

The problem relevant to the lot size determination, for either making or buying decisions, is one crucial issue facing most organizations. One of the oldest pieces of research involving the lot size determination is the work of Harris (1990). In his work, a well-known economics order quantity (EOQ) model was presented to determine the optimal replenishment lot size. The optimal ordered quantity was calculated by taking the first derivative of the annual total cost respected to the ordered quantity. In the past several decades, plenty of research has been developed by relaxing the assumption of the basic EOQ model in several ways. A literature survey on the evolution of Harris's EOQ model was presented by Andriolo, Battini, Grubbström, Persona, and Sgarbossa (2014).

An essential extension of the EOQ model is the case when the system has a finite rate of replenishment. This condition can be found in most production environments. Taft (1918) proposed the economics production quantity (EPQ) model to determine the optimal production quantity by relaxing the assumption of the infinite production rate of the EOQ model. The research assumes that the finite production rate is known and constant. With a finite production rate, the replenishment item is produced and added to the system gradually rather than all at once. The EPQ model and its variants have gained much attention from many researchers. Misra (1975) discussed an EPQ model for a system with a deteriorating inventory. The cost of inventory deterioration was included in the total variable cost. For the case of constant deterioration rate, the optimum production cycle length and production lot size were determined by taking the derivative of total variable cost. A numerical method was suggested to determine the solution when the inventory deterioration rate is Weibull distributed. Darwish (2008) studied the relationship between the setup cost and the production run length in the EPQ model. The research assumes that the setup cost is a function of the production run length defined by a setup cost shape factor. The total cost functions associated with two different cases, EPQ models with and without backorders, were proved to be convex. The optimal solution is, then, determined by separately taking the partial derivative of the total cost function respected to the production cycle length and the backorder level. The result obtained from the numerical experiment shows that the optimal production cycle length increases when the setup cost shape factor approaches zero. Cárdenas-Barrón (2011) presented the basic concepts of analytic geometry and algebra to determine the optimal lot size and backorders level for the system having both linear and fixed backordering costs. The economic production quantity model with backordering cost and imperfect items was discussed by Hayek and Salameh (2001). The model assumes that, for each production cycle, there are two portions of production time: the regular production time and the production time required to rework those nonconforming items occurred. The optimal production lot size and backordering quantity were determined by taking the partial derivative of the expected value of the total cost function. Similar research with multiple shipments was examined by Chiu, Lin, Wu, and Yang

(2011). Unlike the traditional EPQ model, which assumes a continuous inventory issuing policy, the research investigated the realistic case of multiple shipments in each production cycle. The optimal production lot size and number of deliveries are determined by taking partial derivatives of a long-run average total cost function respected to those corresponding decision variables.

For the intermittent production processes, a number of products are generally required to share the same equipment on a rotating basis. Different production items are produced on a regular cycle in predetermined production lot sizes (Tersine, 1994). This situation leads to the research area known as the multiple items EPQ model. Some of those earliest research involving the multiple item EPQ model with the objective of production cycle length determination can be found in the works of Bomberger (1966), Goyal (1973), and Eilon (1985). Taleizadeh, Najafi, and Akhavan Niaki (2010) addressed the problem of multiple items EPQ model with imperfect quality. The defective rate of each item is a random variable and all defective items must be scrapped. Shortages are allowed and fully backordered. The optimal production cycle length and backordered quantities of each item were determined by equating the partial derivatives of the total cost function with zero. The research was later extended by Taleizadeh, Cárdenas-Barrón, and Mohammadi (2014) to the case when some imperfect items can be reworked. Pirayesh and Poormoaied (2015) discussed the research relevant to multiple items EPQ model with production capacity limitation. In order to prevent shortages, those items that cannot be supplied by the production must be procured from outside suppliers. The meta-heuristic based on genetic algorithm and particle swarm optimization is presented to search for a good problem solution. In the situation that many jobs must be processed on the same production line, the determination of proper production sequence is another research area that has gained a lot of attention from many researchers. Comprehensive reviews on the production sequencing and scheduling with job sequence-dependent setup time and/or cost can be found in the works of Allahverdi, Gupta, and Aldowaisan (1999) and Zhu and Wilhelm (2006). Gilmore and Gomory (1964) considered the job sequencing problem on a single machine with sequence-dependent changeover cost as the shortest path traveling salesman problem. Barnes and Vanston (1981) discussed the job sequencing problem for the production system having sequence-dependent setup cost and delay penalty cost. The Dynamic Programming/Branch and Bound (DPBB) algorithm was employed to search for the problem solution. Ángel-Bello, Álvarez, Pacheco, and Martínez (2011) studied a single machine scheduling problem with sequence-dependent setup time and preventive machine maintenance. Here, both time between two consecutive maintenance activities and maintenance time are known and constant. Besides those regular jobs to be processed, each maintenance activity is considered as a job with sequence-dependent maintenance preparing time. The mixed-integer linear model was introduced to determine the solution for the objective of make-span minimization. Dolgui, Kovalyov, and Shchamialiova (2011) presented the multiple items EPQ model with sequence-dependent setup time and imperfect machine. Two different problem cases are considered. While the first one assumes that all product demands are satisfied and the objective is to minimize the make-span, the second one allows some demands to be unsatisfied and has the objective of total dissatisfaction cost minimization.

Traditionally, that research involving the multiple items EPQ model do not consider the effect of production sequence on the total inventory level, which is varied over the time of production. In fact, the maximum value of total inventory level can directly affect the storage space required, a significant constraint concerned by most organizations. Unlike traditional research, this research studies the influence of production sequence on the total inventory level and the optimal production run size. The genetic algorithm is presented to determine the proper production sequence with the objective of minimizing the required storage space.

2. Problem Description

The research study the production system having multiple items being produced on the same production line. Each item has its own setup cost, holding cost, demand rate, and production rate. The shortage is not allowed and, hence, the production rate of each item must be greater than or equal to its demand rate. The total production time required to satisfy the total annual demand of all items must be less than or equal to the available working time per year. The production of all items is conducted in the production cycle. For each production cycle, different items may have different amounts of production quantity to be produced. Nonetheless, each production cycle produces the same amount of each item and has the same production sequence. For any production item, each unit being produced requires one cubic volume of storage space. The research objective is to determine the proper production run size and production sequence in such a way that the total variable cost, composed of the production setup cost and the inventory holding cost, and the required storage space are minimized. The following notations are applied throughout the paper.

n	number of production items
T	production cycle length (years)
m	number of production runs (cycles/year); $m = \frac{1}{T}$
D_i	annual demand of item i (units); for $i = 1, 2, \ldots, n$
$D_{[i]}$	annual demand of item being produced at position i (units); for $i = 1, 2, \ldots, n$
R	total annual demand of all items (units); $R = \sum_{i=1}^{n} D_i$
π_i	annual production rate of item i (units); for $i = 1, 2, \ldots, n$
$\pi_{[i]}$	annual production rate of item being produced at position i (units); for $i = 1, 2, \ldots, n$
H_i	holding cost of item i (dollars per unit per year); for $i = 1, 2, \ldots, n$
S_i	production setup cost (dollars per production setup of each item); for $i = 1, 2, \ldots, n$
Q_i	production quantity of item i (units); for $i = 1, 2, \ldots, n$
$Q_{[i]}$	production quantity of item being produced at position i (units); for $i = 1, 2, \ldots, n$
t_p	production time of item i (years/cycle); for $i = 1, 2, \ldots, n$
$t_{p[i]}$	production time of item being produced at position i (years/cycle); for $i = 1, 2, \ldots, n$
t_{slack}	slack time; $t_{slack} = T - \sum_{i=1}^{n} t_{p[i]}$
j_i	production item i; for $i = 1, 2, \ldots, n$
$j_{[i]}$	production item being produced at position i of the sequence; for $i = 1, 2, \ldots, n$
$I_{[0]}$	total inventory level at the beginning of each production run (units)
$I_{[i]}$	total inventory level after the production of the item being produced at position i is ended (units); for $i = 1, 2, \ldots, n$
I_{max}	maximum value of $I_{[i]}$ (units); for $i = 1, 2, \ldots, n$
$V_{[i]}$	total inventory changing amount from $I_{[i-1]}$ to $I_{[i]}$ (units); $V_{[i]} = I_{[i]} - I_{[i-1]}$; for $i = 1, 2, \ldots, n$
ρ_i	ratio of D_i to π_i; for $i = 1, 2, \ldots, n$
$\rho_{[i]}$	ratio of $D_{[i]}$ to $\pi_{[i]}$; for $i = 1, 2, \ldots, n$
ρ_{slack}	slack proportion which is the ratio between remaining time per production cycle and the production cycle length; $\rho_{slack} = t_{slack}/T$ or $\rho_{slack} = 1 - \sum_{i=1}^{n} \rho_i$
N	number of parent chromosomes
r_x	probability of crossover
r_m	probability of mutation
F_p	fitness value of the chromosome p; for $p = 1, 2, \ldots, 2N$
P_p	probability that the chromosome p is selected; for $p = 1, 2, \ldots, 2N$

The problem characteristics can be illustrated as shown in example 1.

Example 1: Given that there are five items to be produced on the same production line. The information regarding annual demand, annual production rate, holding cost rate, and setup cost of each production item is demonstrated in Table 1. Here, the total available number of working days per year is assumed to be 250.

Item i	D_i	π_i	H_i	S_i
1	5,000	25,000	1.60	40
2	10,000	100,000	1.40	25
3	7,000	87,500	0.60	30
4	15,000	50,000	1.15	27
5	4,000	25,000	1.65	80

Table 1. Parameters of each production item

One possible solution is to have ten production runs in a year (m = 10) with the production sequence of 1→3→5→2→4 for each production run (note that this solution may not be optimal). The production quantity of each item (Q_i) in each production cycle can be calculated by dividing the annual demand by the annual number of production runs. The production time of each item (t_{pi}) for each production cycle can be determined by dividing its production quantity by its annual production rate. The result of production quantity and production time of each item, in each production cycle, can be calculated as shown in Table 2.

Item i	Q_i (units)	t_{pi} (years)
1	5,000/10 = 500	500/25,000 = 0.02
2	10,000/10 = 1,000	1,000/100,000 = 0.01
3	7,000/10 = 700	700/87,500 = 0.008
4	15,000/10 = 1,500	1,500/50,000 = 0.03
5	4,000/10 = 400	400/25,000 = 0.016

Table 2. Production quantity and production time of each item for each cycle

The total annual system cost is the summation of annual setup cost and annual inventory holding cost of all items, as shown in equation (1).

$$TC(m) = \sum_{i=1}^{n} mS_i + \sum_{i=1}^{n} \frac{H_i D_i (\pi_i - D_i)}{2m\pi_i} \tag{1}$$

According to equation (1), the annual setup cost and the annual inventory holding cost of the solution mentioned earlier can be calculated as shown in Table 3. Here, the total annual cost is 2,020.00 + 2,024.15 = 4,044.15 dollars.

Item i	Annual setup cost (dollars)	Annual holding cost (dollars)
1	(10)(40) = 400.00	$\frac{(1.60)(5{,}000)(25{,}000 - 5{,}000)}{(2)(10)(25{,}000)}$ = 320.00
2	(10)(25) = 250.00	$\frac{(1.40)(10{,}000)(100{,}000 - 10{,}000)}{(2)(10)(100{,}000)}$ = 630.00
3	(10)(30) = 300.00	$\frac{(0.60)(7{,}000)(87{,}500 - 7{,}000)}{(2)(10)(87{,}500)}$ = 193.20
4	(10)(27) = 270.00	$\frac{(1.15)(15{,}000)(50{,}000 - 15{,}000)}{(2)(10)(50{,}000)}$ = 603.75
5	(10)(80) = 800.00	$\frac{(1.65)(4{,}000)(25{,}000 - 4{,}000)}{(2)(10)(25{,}000)}$ = 277.20
Total	2,020.00	2,024.15

Table 3. Annual setup cost and annual holding cost of each production item

Consider the production sequence of 1→3→5→2→4 and the production time of each item (t_{pi}) calculated in Table 2, the inventory level of each item and the total inventory level can be illustrated as shown in Figure 1. It can be seen that the maximum total inventory level occurs after the production of the item being produced at the last position ($I_{[5]} = I_4$) is finished. Here, the required storage space should be assigned large enough to keep all 2,158 units, the maximum total inventory level.

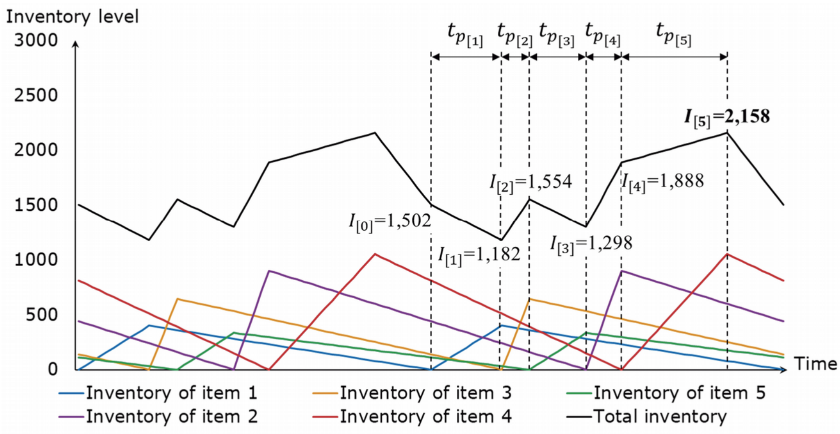

Figure 1. Inventory levels for the production sequence of 1→3→5→2→4

End of example

3. Problem Analysis

3.1. The Effect of Production Sequence on Maximum Inventory Level

Given that there are n items to be produced in each production cycle, the total inventory level, $I_{[i]}$ (i = 1, 2, …, n), is the sum of inventory levels of all items after the production of the item being produced at the position i is ended. Here, the ($I_{[0]}$ is defined as the total inventory level at the beginning of each production run. The maximum inventory level, I_{max}, is the maximum value of $I_{[0]}$, $I_{[1]}$, $I_{[2]}$, …, $I_{[n]}$. Note that the storage space required should be set equal to the maximum inventory level. Consider the changes in the amount of total inventory level from $I_{[i-1]}$ to $I_{[i]}$ as $V_{[i]}$, the relationship between the $V_{[i]}$ and $I_{[i]}$ for the system having five production items is demonstrated as shown in Figure 2.

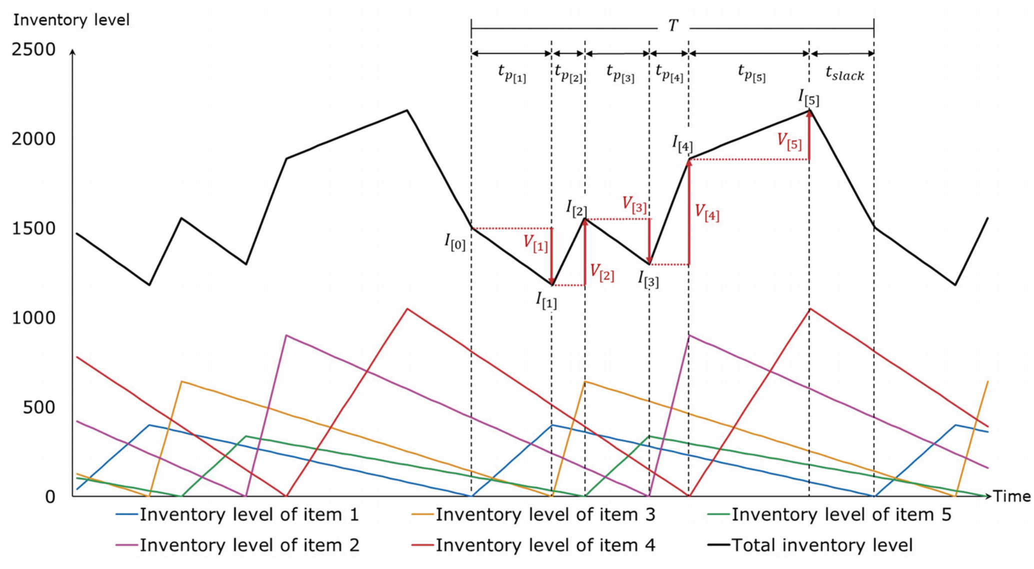

Figure 2. Inventory levels for the production sequence of 1→3→5→2→4

From Figure 2, there are five items to be produced on the same production line with the sequence of $j_{[1]} = j_1$, $j_{[2]} = j_3$, $j_{[3]} = j_5$, $j_{[4]} = j_2$, and $j_{[5]} = j_4$. The relationship between the $I_{[i]}$ and the $V_{[i]}$ can be written as follows.

$$I_{[i]} = I_{[i-1]} + V_{[i]} \qquad \text{for } i = 1, 2, \dots, n \tag{2}$$

According to equation (2), the calculation of $I_{[i]}$ relevant to the $I_{[0]}$ is shown in equation (3).

$$I_{[i]} = I_{[0]} + \sum_{j=1}^{i} V_{[j]} \qquad \text{for } i = 1, 2, \dots, n \tag{3}$$

Consider Figure 2, the value of $I_{[0]}$ can be calculated as shown below.

$$I_{[0]} = D_{[2]}\left(t_{p_{[1]}}\right) + D_{[3]}\left(t_{p_{[1]}} + t_{p_{[2]}}\right) + \dots + D_{[n]}\left(t_{p_{[1]}} + t_{p_{[2]}} + \dots + t_{p_{[n-1]}}\right)$$
$$I_{[0]} = \sum_{j=2}^{n}\left(D_{[j]}\sum_{k=1}^{j-1} t_{p_{[k]}}\right) \tag{4}$$

The $t_{p[k]}$, the time required to produce the item placed at the position k of the production sequence can be calculated as follows.

$$t_{p_{[k]}} = \frac{Q_{[k]}}{\pi_{[k]}} \qquad \text{for } k = 1, 2, \dots, n \tag{5}$$

The changes in the amount of total inventory level ($V_{[i]}$) is the increasing amount of the item produced during the production time ($t_{p[i]}$) subtracted by the demand of other items during that time. The calculation can be shown as follows.

$$V_{[i]} = \left(\pi_{[i]} - D_{[i]}\right)t_{p_{[i]}} - \sum_{\forall j \neq i} D_{[j]}\, t_{p_{[i]}}$$
$$V_{[i]} = \left(\pi_{[i]} - R\right)t_{p_{[i]}} \tag{6}$$

The following example illustrates the use of equations (3), (4), and (6) to determine the total inventory levels of the problem mentioned in example 1.

Example 2: Given the data provided in Tables 1 and 2 of example 1, the value of $I_{[0]}$ can be calculated using equation (4) as follows.

$I_{[0]} = (7{,}000)(0.02) + (4{,}000)(0.02 + 0.008) + (10{,}000)(0.02 + 0.008 + 0.016) + (15{,}000)(0.02 + 0.008 + 0.006 + 0.01)$

$I_{[0]} = 1{,}502$ units

Reveal the data related to the production rate, demand rate, and production time of each item in example 1, the values of total inventory level changing amount ($V_{[1]}$ to $V_{[5]}$) can be determined using equation (6) as follows.

$V_{[1]} = (25{,}000 – 41{,}000)(0.02) = –320$ units

$V_{[2]} = (87{,}500 – 41{,}000)(0.008) = 372$ units

$V_{[3]} = (25{,}000 – 41{,}000)(0.016) = –256$ units

$V_{[4]} = (100{,}000 – 41{,}000)(0.01) = 590$ units

$V_{[5]} = (50{,}000 – 41{,}000)(0.03) = 270$ units

Applying equation (3), the values of total inventory levels ($I_{[1]}$ to $I_{[5]}$) can, then, be calculated as shown below.

$I_{[1]} = 1{,}502 + (–320) = 1{,}182$ units

$I_{[2]}$ = 1,502 + (–320 + 372) = 1,554 units

$I_{[3]}$ = 1,502 + (–320 + 372 – 256) = 1,298 units

$I_{[4]}$ = 1,502 + (–320 + 372 – 256 + 590) = 1,888 units

$I_{[5]}$ = 1,502 + (–320 + 372 – 256 + 590 + 270) = 2,158 units

Consider all total inventory levels ($I_{[0]}$ to $I_{[5]}$), the maximum value occurs at $I_{[5]}$ = 2,158 units. Here, for the production sequence of 1→3→5→2→4, the storage space should be designed such that it can keep the maximum inventory level of 2,158 units.

End of example

In fact, different production sequences can result in different storage spaces required. Figure 3 illustrates two different production sequences of the problem mentioned in example 1. Note that the maximum total inventory levels obtained from both production sequences are not the same. While the maximum total inventory level for the production sequence of 1→3→5→2→4 is 2,158 units, the maximum total inventory level for the production sequence of 2→1→3→5→4 is 1,878 units. Here, the optimal production sequence should be the one that yields the minimum value of the maximum total inventory level.

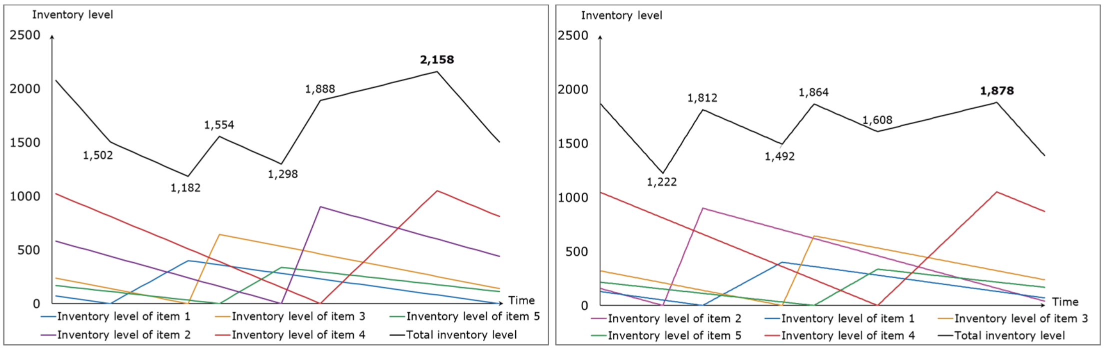

Figure 3. Inventory levels of the example problem for two different sequences

3.2. The Relationship between Optimal Production Sequence and Number of Production Runs

To determine the relationship between the optimal production sequence and the number of production runs, replace the value of $I_{[0]}$ and $V_{[i]}$ from equations (4) and (6), respectively, to equation (3). The result is shown in the following equation.

$$I_{[i]} = \sum_{j=2}^{n}\left(D_{[j]}\sum_{k=1}^{j-1} t_{p_{[k]}}\right) + \sum_{l=1}^{i}(\pi_{[l]} - R)t_{p_{[l]}} \qquad \text{for } i = 1, 2, \ldots, n \tag{7}$$

The production time of the item processed at the position i, $t_{p[i]}$, is the value of the production quantity of that item divided by its production rate. Knowing that the production quantity is the annual demand divided by the number of production runs in a year, the relationship between the production time of the item processed at the position i and the number of production runs can be determined as shown in equation (8).

$$t_{p_{[i]}} = \frac{Q_{[i]}}{\pi_{[i]}} = \frac{D_{[i]}}{m\pi_{[i]}}$$

$$t_{p_{[i]}} = \frac{\rho_{[i]}}{m} \tag{8}$$

Replacing equation (8) in equation (7), the calculation of each $I_{[i]}$ can be rearranged as shown in equation (9).

$$I_{[i]} = \frac{1}{m}\left[\sum_{j=2}^{n}\left(D_{[j]}\sum_{k=1}^{j-1}\rho_{[k]}\right) + \sum_{l=1}^{i}(\pi_{[l]} - R)\rho_{[l]}\right] \quad (9)$$

Consider equation (9), the value of total inventory level can be decreased (or increased) according to the multiplication factor $1/m$. Here, when there are m production cycles in a year, the total inventory level of any item in the sequence can be calculated as $1/m$ times the inventory level of the item when there is only one production cycle per year. Therefore, it can be concluded that the optimal production sequence is independent of the number of production runs.

4. Methodology

From the previous section, the optimal production sequence is independent of the number of production runs. In other words, determining the proper solution for the research problem can be divided into two independent subproblems. The first subproblem is to determine the optimal number of production runs in such a way that the total system variable cost composing of inventory holding cost and setup cost is minimized. The second subproblem is to search for the optimal production sequence with the objective of minimizing the required storage space.

In order to determine the solution for the first subproblem, the traditional multi-item economics production quantity model may be applied. Taking the derivative of equation (1) respected to the number of production runs, the optimal number of production runs can be determined as shown in equation (10). Equation (11) demonstrates the calculation of the production quantity of each item at the specific number of production runs.

$$m^* = \sqrt{\frac{\sum_{i=1}^{n}\frac{H_i D_i(\pi_i - D_i)}{\pi_i}}{2\sum_{i=1}^{n} S_i}} \quad (10)$$

$$Q_i^* = \frac{D_i}{m^*} \quad (11)$$

Since the optimal production sequence is independent of the number of production runs per year, to determine the optimal production sequence for the first subproblem, the production cycle length of $T = 1$ year may be considered. Note that this is the situation when there is only one production cycle in a year. With the value of $m = 1$, equation (9) can be rewritten as shown in equation (12).

$$I_{[i]} = \sum_{j=2}^{n}\left(D_{[j]}\sum_{k=1}^{j-1}\rho_{[k]}\right) + \sum_{l=1}^{i}(\pi_{[l]} - R)\rho_{[l]} \quad (12)$$

The research presents the genetic algorithm to determine a proper production sequence in a reasonable amount of time. The procedure is to, with the assumption of one production cycle in a year, search for a proper production sequence via the evolutionary mechanism. Here, equation (12) is integrated to determine the total inventory level by the time that the production of each item in the sequence is finished. Then, the concept of multiple items EPQ model is applied to determine the optimal number of production runs per year, as shown in equation (10). Finally, with the optimal number of production runs per year, equation (9) can be applied to calculate the total inventory level after the production of the item being produced at the position i is ended.

5. Genetic Algorithm

The genetic algorithm is a powerful metaheuristic that imitates the biological evolutionary concept to search for a proper solution to the problem. It is suitable to be applied to the problem with a large size possible solution set. Figure 4 illustrates the structure of the genetic algorithm employed in this research.

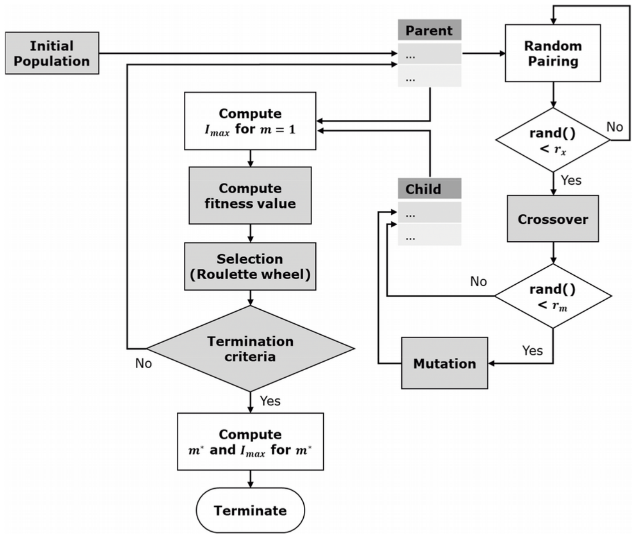

Figure 4. Structure of the genetic algorithm

5.1. Chromosome Representation and Initialization

A chromosome, a one-dimensional array of the integers, is used to represent a production sequence. There are *n* genes in each chromosome. Each gene represents the job being processed at each position. As an example of the chromosome representation, the array of [1 3 5 2 4] represents a production sequence that starts from item 1 and follows by item 3, item 5, item 2, and item 4, respectively. Here, the initial population is randomly created from the permutation of genes until the number of initial chromosomes is equal to the predetermined population size.

5.2. Crossover

A crossover is an operator being used to exchange information between a pair of parent chromosomes "Parent 1" and "Parent 2". As claimed by Lee and Choi (1995) and Supithak and Plongon (2011), the uniform order-based crossover is proper to implement with the job sequencing problem. The research is, therefore, select this technique as the genetic crossover operator. In order to conduct the uniform order-based crossover, a pair of different chromosomes is randomly selected from the parent population. An array of binary numbers (0, 1) with the same size as the parent chromosome is generated. The parent chromosomes exchange their genes according to the array to create two offspring. The mechanism of the uniform order-based crossover is illustrated as shown in Figure 5.

For each parent selection, the chance that the crossover will occur is equal to the r_x, the probability of crossover. In order to make this happen, a random number (0, 1) is generated. The crossover will be applied only when the random number value is less than or equal to the r_x. The crossover process must recursively occur until the number of generated children equals the number of parents.

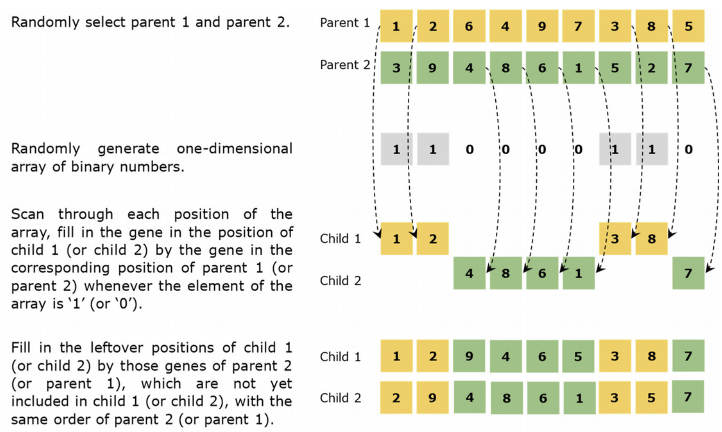

Figure 5. Illustration of the uniform ordered-based crossover procedure

5.3. Mutation

A mutation is a significant operator that operates on the child chromosome of the latest generation to create some mutant children and, hence, increase the population variety. In the proposed genetic algorithm, the random exchanging mutation is implemented. Two positions of a chromosome are randomly selected and interchange their genes. Each child chromosome has the chance of r_m to be mutated. Here, a random number (0, 1) is generated. The mutation will occur only when the value of the random number is less than or equal to the r_m. Some child chromosomes are still nonmutant.

5.4. Evaluation

The evaluation process aims to measure the quality of each chromosome, simply called the fitness value (F_p). Since the objective of production sequencing is to minimize the maximum inventory level, a chromosome having a low value of maximum inventory level should have a high fitness value. Equation (13) demonstrates the calculation of the fitness value.

$$F_p = \frac{1}{I_{max}} \tag{13}$$

5.5. Selection

A roulette wheel selection method is applied to the proposed GA to imitate the natural selection. Those better suit chromosomes should be selected as members of the next generation. According to the roulette wheel selection method, a circle is divided into 2N pies of unequal size. Each piece of the pie represents the chromosome in the considered generation. The size of each pie is related to the chromosome's fitness. A better chromosome should have a higher probability of being chosen. The probability that the p^{th} chromosome is selected can be calculated as shown in equation (14).

$$P_p = \frac{F_p}{\sum_{k=1}^{2N} F_k} \tag{14}$$

After calculating the values of probability of selection, all chromosomes must be reindexed in ascending order of their selection probability values. In order to simulate a roulette wheel spinning, the random number (*rand*( )) within the range of [0, 1] is randomly generated. This random number represents the cumulative probability. The first chromosome satisfying the condition of $\sum_{p=1}^{i} P_p \geq rand()$ is selected as a member of the next generation. Here, the roulette wheel must be spun for N times to create N parent chromosomes of the next generation.

5.6. Parameter Setup

To determine the proper parameters, the GA is run on 30 problems of 10 production items at three levels of crossover rate (r_x = 0.7, 0.8, 0.9), two levels of mutation rate (r_m = 0.1, 0.2), and two levels of population size (2N = 1000, 2000). It was found that, among all 3 × 2 × 2 = 12 combinations, the combination of (r_x = 0.8, rm = 0.1, and 2N = 2000) yields the best result. In comparison to the optimal solution searched by the enumeration method, this combination provides the optimal solution for 28 out of 30 problems and, therefore, it has been chosen for further study.

6. Experimental Result

Two numerical experiments are conducted to evaluate the performance of the proposed GA. The first experiment is to compare the solutions obtained from the GA, the Largest Demand First (LDF) heuristic, the Largest Pi First (LPF) heuristic, and the Largest Rho First (LRF) heuristic with the optimal solution determined by the enumeration method on 360 small size problems. From the first experiment, the best heuristic among all threes (LDF, LPF, and LRF) is selected for further study. The second experiment is to compare the solution yielded from the GA with the solution acquired by the selected heuristic on 180 large size problems. The proposed GA is coded using C programming language in the CodeBlocks IDE version 20.03 and run on the Intel Xeon Six-Core CPU. The proposed GA is terminated by two criteria. For the first criterion, the GA is terminated when the improvement after 300 consecutive iterations is less than 0.01 percent. The second criterion stops the search when the number of generations reaches 10,000. Here, the GA is terminated when any of the two criteria is met.

For the numerical experiment setup, the unit cost of each item is randomly generated from a discrete uniform random number of [100, 200]. The holding cost fraction is randomly chosen from a discrete uniform random number of [0.15, 0.30] (with 0.01 unit of incremental value). The annual demand of each item is randomly selected from a discrete uniform random number of [5000, 20000]. The annual production rate of each item is determined according to the ratio between the annual demand rate and the annual production rate, ρ_i. Here, the ρ_i is randomly generated in such a way that it has a value between 0 and 1, and the summation of all ρ_i in each problem must conform to the definition of slack proportion ($\rho_{slack} = 1 - \sum_{i=1}^{n} \rho_i$). For the first experiment, the three factors to be evaluated are the number of production items (8 items, 10 items), the setup cost to holding cost ratio (10, 20), and the slack proportion (0.2, 0.4, 0.6). The second experiment is conducted on 15 production items and, hence, the remaining two factors to be evaluated are the setup cost to holding cost ratio (10, 20) and the slack proportion (0.2, 0.4, 0.6). The setup parameters are shown in Table 4.

There are 2 × 2 × 3 = 12 treatment combinations in the first experiment. With 30 replications of each treatment combination, there are totally 360 randomly generated problems. This experiment aims to evaluate the performances of the proposed GA and three other heuristics: the LDF heuristic, the LPF heuristic, and the LRF heuristic. Here, the LDF, LPF, and LRF heuristics are to arrange the production sequence according to the non-increasing order of the demand rate (D_i), the production rate (π_i), and the ratio between demand rate and production rate (ρ_i), respectively. The experiment is conducted by comparing the solutions yielded from the GA, LDF heuristic, LPF heuristic, and LRF heuristic with the optimal solution obtained from the enumeration method. For the numerical comparison, the percentage deviations between the solution yielded from each considered method and the optimal solution ($\%Dev_{GA-OPT}$, $\%Dev_{LDF-OPT}$, $\%Dev_{LPF-OPT}$, and $\%Dev_{LRF-OPT}$) are calculated as shown in the following equations.

Characteristics	Value
Number of production items (n)	Experiment 1: 8 items, 10 items Experiment 2: 15 items
Unit cost (C_i)	Discrete uniform [100, 200]
Holding cost fraction (h_i)	Discrete uniform [0.15, 0.30]
Holding cost of item (H_i)	$h_i C_i$
Setup cost (S_i) to holding cost (H_i) ratio	(10, 20)
Annual demand (D_i)	Discrete uniform [5000, 20000]
Annual production rate (π_i)	D_i/ρ_i
Ratio between D_i and π_i (ρ_i)	$[rand() \times (1 - \rho_{slack})]/\sum_{i=1}^{n} \rho_i$
Slack proportion (ρ_{slack})	0.2, 0.4, 0.6

Table 4. Setup parameters for the experimental problems

$$\% Dev_{GA-OPT} = \frac{I_{max}^{GA} - I_{max}^{OPT}}{I_{max}^{OPT}} \times 100 \quad (15)$$

$$\% Dev_{LDF-OPT} = \frac{I_{max}^{LDF} - I_{max}^{OPT}}{I_{max}^{OPT}} \times 100 \quad (16)$$

$$\% Dev_{LPF-OPT} = \frac{I_{max}^{LPF} - I_{max}^{OPT}}{I_{max}^{OPT}} \times 100 \quad (17)$$

$$\% \, Dev_{LRF-OPT} = \frac{I_{max}^{LRF} - I_{max}^{OPT}}{I_{max}^{OPT}} \times 100 \quad (18)$$

Where,

I_{max}^{OPT} is the maximum inventory of the optimal solution obtained from the enumeration method.

I_{max}^{GA} is the maximum inventory of the solution obtained from the GA.

I_{max}^{LDF} is the maximum inventory of the solution obtained from the LDF heuristic.

I_{max}^{LPF} is the maximum inventory of the solution obtained from the LPF heuristic.

I_{max}^{LRF} is the maximum inventory of the solution obtained from the LRF heuristic.

The result on the 95% confidence intervals on mean of percentage deviation, the number of optimal solutions found (from 360 problems), and the average computational times obtained from the GA, LDF, LPF, and LRF is shown in Table 5.

According to the result obtained from the first experiment, the proposed GA performs better than the other three heuristics. Here, the GA provides the optimal solutions for 340 out of 360 problems. The 95% confidence interval on mean of the percentage deviation is (0.0015, 0.0123). The average computational time for the GA is 1.69 seconds. Besides the proposed GA, among all three heuristics (LDF, LPF, and LRF), the LPF heuristic yields the preferable solution. The LPF delivers the optimal solutions for 78 out of 360 problems. The 95% confidence interval on mean of the percentage deviation is (4.7084, 5.7989).

Method	Percentage deviation (95% confidence interval)	Number of optimal solutions found(from 360 problems)	Average computational time
GA	(0.0015, 0.0123)	340	1.69 sec.
LDF	(10.0365, 11.5154)	0	< 0.1 sec.
LPF	(4.7084, 5.7989)	78	< 0.1 sec.
LRF	(26.0249, 28.0284)	0	< 0.1 sec.

Table 5. Result of 95% confidence intervals on mean of percentage deviation, number of optimal solutions found, and average computational times

For further analysis of the first experiment, the nonparametric statistical tests are conducted to determine the main effect of the three factors (the number of production items, the setup cost to holding cost ratio, and the slack proportion) on the performance of the proposed GA. The results are shown below.

Consider the result obtained from the Mann-Whitney test, at the 95 percent confidence interval, while there is no statistically significant difference between the two levels of setup cost to holding cost ratio factor (*p*-value = 0.6663), the main effect of the number of production items factor is significant (*p*-value = 0.0000). Here, out of 180 experimental problems conducted in the first experiment, the number of optimal solutions found when there are 8 and 10 production items are 179 and 161, respectively. The Kruskal-Wallis test result indicates significant differences at different levels of the slack proportion factor (*p*-value = 0.000). Consider the number of optimal solutions found at each level of the slack proportion factor (ρ_{slack}), out of 120 experimental problems at each level, the number of optimal solutions found at the values of ρ_{slack} of 0.2, 0.4, and 0.6 are 100, 120, and 120, respectively. It can be inferred that the performance of the proposed GA is increased as the value of ρ_{slack} is getting larger.

The second experiment is conducted on the large size problem of 15 production items. The experiment aims to study the main effects of two factors: the setup cost to holding cost ratio (10, 20) and the slack proportion (0.2, 0.4, 0.6). There are totally 2 × 3 = 6 treatment combinations. With 30 replications for each treatment, 180 experimental problems are randomly generated. Here, from the result of the first experiment, the LPF heuristic outperforms the

LRF and LDF heuristics. With the small value of percentage deviation mean, having a 95% confidence interval of (4.7084, 5.7989) in the first experiment, the LPF is selected as the benchmark to compare with the GA in the second experiment. The percentage deviation between the solution obtained from the GA and the solution yielded from the LPF heuristic is considered as the response variable and can be calculated as shown in equation 19.

According to the experimental result, in comparison to the LPF heuristic, the GA provides a better solution for 158 out of 180 problems. For the remaining 22 problems, both GA and LPF yield the same solutions. The 95% confidence interval on mean of the percentage deviation is (5.5629, 7.0435). The average and maximum values of percentage deviation are 6.3032 and 17.6680 percent, respectively. Three statistical nonparametric techniques, the Mann-Whitney, the Kruskal-Wallis, and the Wilcoxon signed rank tests, are conducted to determine the main effect of each factor on the GA performance. The test results are shown in Figures 7 and 8.

From Figure 7, the Mann-Whitney statistic is 8148.5, and the associated *p*-value is 0.9931. It can be concluded that, at 95 percent confidence level, there is no statistically significant difference between the two levels of the setup cost to holding cost ratio factor. According to the result obtained from the Wilcoxon signed rank test, the two confidence intervals have their ranges overlap to each other. This condition emphasizes the result concluded by the Mann-Whitney test.

To evaluate the main effect of the slack proportion (ρ_{slack}) factor, the Kruskal-Wallis test has been conducted. As shown in Figure 8, at 95 percent confidence level, the values of test statistic and *p*-value are 137.74 and 0.000, respectively. It can be concluded that at least one level of the factor has a different median from the other two. The inequality is further examined by the Wilcoxon signed rank test. The result shows that the three confidence intervals for the population medians do not overlap with each other. This situation indicates that there are significant differences among all three levels of the factor. According to the confidence intervals, in comparison to the LPF heuristic, the GA is more outstanding at smaller values of slack proportion.

```
Mann-Whitney Test and CI: Number of production items(n) = 10, 8
        N   Median
n=10  180  0.00000
n=8   180  0.00000
Point estimate for η1 - η2 is 0.00000
95.0 Percent CI for η1 - η2 is (0.00000,0.00000)
W = 30863.5
Test of η1 = η2 vs η1 ≠ η2 is significant at 0.0996
The test is significant at 0.0000 (adjusted for ties)

Mann-Whitney Test and CI: Setup cost to holding cost ratio = 10, 20
           N   Median
S = 10H  180  0.00000
S = 20H  180  0.00000
Point estimate for η1 - η2 is -0.00000
95.0 Percent CI for η1 - η2 is (0.00000,-0.00000)
W = 32320.5
Test of η1 = η2 vs η1 ≠ η2 is significant at 0.8641
The test is significant at 0.6663 (adjusted for ties)

Kruskal-Wallis Test: %Dev_GA-OPT versus Slack proportion
Kruskal-Wallis Test on %Dev_GA-OPT
Slack proportion        N       Median  Ave Rank      Z
0.2                   120  0.000000000     200.5   2.58
0.4                   120  0.000000000     170.5  -1.29
0.6                   120  0.000000000     170.5  -1.29
Overall               360                  180.5
H = 6.65   DF = 2  P = 0.036
H = 42.19  DF = 2  P = 0.000  (adjusted for ties)
```

Figure 6. Nonparametric test results of the first experiment

$$\% \, Dev_{GA-LPF} = \frac{I_{max}^{LPF} - I_{max}^{GA}}{I_{max}^{GA}} \times 100 \quad (19)$$

```
Mann-Whitney Test and CI: Si/Hi = 10, 20
              N   Median
Si/Hi = 10  90   6.230
Si/Hi = 20  90   6.241
Point estimate for η1 - η2 is 0.000
95.0 Percent CI for η1 - η2 is (-1.346,1.345)
W = 8148.5
Test of η1 = η2 vs η1 ≠ η2 is significant at 0.9932
The test is significant at 0.9931 (adjusted for ties)

Wilcoxon Signed Rank CI: Si/Hi = 10, 20
                    Estimated    Achieved   Confidence Interval
               N     Median    Confidence     Lower     Upper
Si/Hi = 10   90       6.14          95.0       4.99      7.34
Si/Hi = 20   90       6.12          95.0       4.98      7.32
```

Figure 7. Nonparametric test result for the main effect of S_i/H_i factor

```
Kruskal-Wallis Test: %DevGA-LPF versus Slack proportion
Kruskal-Wallis Test on %DevGA-LPF
Slack proportion     N    Median  Ave Rank       Z
0.2                 60   10.9088     142.7    9.49
0.4                 60    6.4245      97.2    1.21
0.6                 60    0.3989      31.7  -10.71
Overall            180                90.5
H = 137.49  DF = 2  P = 0.000
H = 137.74  DF = 2  P = 0.000  (adjusted for ties)

Wilcoxon Signed Rank CI: Slack proportion = 0.2,0.4,0.6
                              Estimated     Achieved     Confidence Interval
                          N     Median    Confidence       Lower     Upper
Slack proportion = 0.2   60     11.43          95.0        10.42     12.35
Slack proportion = 0.4   60      6.46          95.0         5.82      7.21
Slack proportion = 0.6   60     0.787          95.0        0.448     1.125
```

Figure 8. Nonparametric test result for the main effect of slack proportion factor

7. Conclusion

The research is to determine the proper production sequence and the number of production runs in the system composing of multiple items being produced on the same production line. While the objective of a number of production runs determination is to reduce the total variable cost composed of inventory holding cost and production setup cost, the production sequence should be arranged in such a way that the storage space required is minimized. It has been found that the two subproblems are independent of each other and can be solved separately. For the first subproblem, the determination of optimal number of production runs, the traditional multiple items EPQ model can still be applied. In order to search for the proper solution of the second subproblem, the production sequencing, the genetic algorithm (GA) is proposed. The calculation of the maximum total inventory level for each production sequence is presented and integrated with the proposed GA. For the performance evaluation of the proposed GA, two experiments are conducted. The first experiment is to compare the solution obtained from the GA with the optimal solution for the small size problem. Three factors are to be investigated: number of items (2 levels: 8, 10), setup cost to holding cost ratio (2 levels: 10, 20), and slack proportion (3 levels: 0.2, 0.4, 0.6). The result shows that the GA can provide the optimal solution for 340 out of 360 randomly generated problems. The 95% confidence interval on mean of the percentage deviation is (0.0015, 0.0123). The main effects of the setup cost to holding cost ratio factor is not found to be significant on the GA performance. The other two factors, the number of production items and the slack proportion, are statistically significant. The performance of the proposed GA is increased at lower and higher levels of the number of production items and the slack proportion, correspondingly. The second experiment is conducted on those large size problems of 15 production items. The objective is to study the main effects of setup cost to holding cost ratio (2 levels: 10, 20) and slack proportion (3 levels: 0.2, 0.4, 0.6) factors. The solution obtained from the GA is compared with the solution yielded from the Largest Pi First (LPF) heuristic. The result indicates that the GA provides the better solution for 158 out of 160 experimental problems. The 95% confidence interval on mean of the percentage deviation is (5.5629, 7.0435). The average and maximum percentage deviations are 6.3032 and 17.6680, respectively. While the main effect of setup cost to holding cost ratio is not remarkable, the main effect of slack proportion is found to be significant. In comparison to the LPF heuristic, the GA performance is more pronounced as the value of slack proportion is decreased.

Declaration of Conflicting Interests

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