OmniaScience

Journal of Industrial Engineering and Management
JIEM, 2022 – 15(4): 663-669 – Online ISSN: 2013-0953 – Print ISSN: 2013-8423
https://doi.org/10.3926/jiem.4078

Optimal Shape for a Rectangular Warehouse with a Lateral Receive/Ship Dock

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Received: May 2022
Accepted: September 2022

Abstract:

Purpose: This technical note provides the mathematical demonstration for obtaining the optimal aspect ratio for a rectangular storage area with a lateral receive/ship dock, representing the standard configuration of modern distribution centers and logistic warehouses. The proposed aspect ratio is the one that minimizes the travel times of operators, keeping the common assumption of a storage area having a uniform access probability.

Design/methodology/approach: To obtain the optimal aspect ratio of the storage area, the entry point of the uniformly distributed dock is modeled with a random variable, with a continuous uniform distribution, and the average travel path of the operator is consequently assessed as a function of the latter. Successively, the average roundtrip length of the operator is evaluated and minimized, leading to the optimal aspect ratio of the storage area.

Findings: It is found that the optimal aspect ratio between the warehouse width (U) and length (V) equals 1.5. The obtained result – which is novel for the warehouse design research since no other contribution discussed it – shows that the operators' travel times are minimized with a storage area where 1 = 1.5V.

Research limitations/implications: Warehouses with a dock on one side now represent the standard configuration of modern distribution centers'. However, no optimal aspect ratio for the storage area has been discussed. For this reason, the paper fills this lack of scientific literature in the warehouse optimization research by providing indications on how to design this class of warehouses.

Practical implications: Distribution managers find here guidance for defining a proper design of logistics centers and evaluating the operators' 'travel times to perform a roundtrip within the storage area. Indeed, the found aspect ratio indicates the optimal design for a storage area with the abovementioned assumptions.

Originality/value: Traditional warehouse shape optimization models assume a single input/output point to the storage area. To our knowledge, no formal demonstration has been proposed for a warehouse with a dock on one entire side. Hence, the added value and novelty of this contribution are given by the possibility to adopt the optimal aspect ratio to perform an adequate design of distribution centers and warehouses.

Keywords: warehouse design and layout, optimization, material handling, travel distance, aspect ratio, logistics

To cite this article:

Di Luozzo, S., & Schiraldi, M.M. (2022). Optimal shape for a rectangular warehouse with a lateral receive/ship dock. *Journal of Industrial Engineering and Management*, 15(4), 663-669. https://doi.org/10.3926/jiem.4078

1. Introduction

The optimal shape of a rectangular storage area is a popular topic in the warehouse design field, and its development has been rapidly spreading in the last decades (Fumi, Scarabotti & Schiraldi, 2013; Gallego, Shaw & Simchi-Levi, 1992; Lee & Elsayed, 2005; Mirzaei, Zaerpour & de Koster, 2021). The first scientific contribution providing an estimation of the optimal design for a rectangular storage area in a warehouse is the one proposed by Francis (1967), which analyses a warehouse with a single input/output point and a storage area with a uniform probability of being accessed by the operators. Specifically, the paper reports the optimal shape of this class of warehouses, referred as the ratio between the length and width of the storage area.

Moreover, other research works have developed models addressing various instances of the warehouse design problem (Ashayeri & Gelders, 1985; De Koster, Johnson & Debjit, 2017; Ekren, 2021), including further characterizations of the warehouse (Bassan, Roll & Rosenblatt, 1980), optimal picking strategies (Litvak, 2006; Litvak & Adan, 2002) or response time considerations (Pandit & Palekar, 1993). However, it seems that no contribution in literature has addressed the problem of optimizing the layout in a warehouse with input/output operations performed on one entire side. Indeed, a brief mention of this problem is reported in the book of Caron, Wegner and Marchet, (1997). Still, no formal demonstration of the optimal aspect ratio is presented, leaving decision-makers and warehouse managers with ambiguity when addressing this specific instance. For this reason, the following research question is addressed in this research article:

RQ: Which is the optimal aspect ratio (i.e. ratio between width and length of a distribution center) that minimizes the travel times of operators for a rectangular storage area with a lateral receive/ship dock?

Therefore, this short technical note provides the mathematical demonstration for obtaining the optimal aspect ratio for a rectangular storage area with a uniformly distributed input/output side corresponding to a lateral receive/ship dock, representing the standard configuration of modern distribution centers and logistic warehouses. The proposed aspect ratio is the one that minimizes the travel times of operators, keeping the assumption of Francis (1967) of a storage area having a uniform access probability.

The height of the warehouse and its storage system configuration (racks, stacks, etc.) are not of interest, being the model valid and appliable independently from them. As a further development of this research work, the presented method can be easily adapted to other warehouse optimization problems and may be exploited for further advancements in the warehouse design field.

2. Background

Warehouse optimization problems are one of the most investigated research streams within the supply chain research field, whose industrial applications range from the optimization of warehouse routing (Pan, Shih & Wu, 2012) to the definition of ideal storage policies (Roodbergen, Sharp & Vis, 2008), to the minimization of traveling times of operators (Fumi et al., 2013) and to the warehouse layout definition (De Koster et al., 2017). Within the latter class of research works, the scientific contribution proposed by Francis (1967) is the first one to study the rectangular warehouse design and layout, with the following assumptions:

- The warehouse is rectangular;
- The height and area of the warehouse are predetermined;
- A single input/output point is considered;
- A storage area with a uniform probability of being accessed by the operators is adopted, while the items may have a different turnover.

By considering these characteristics, the paper of Francis (1967) provides an estimation of the optimal shape for different rectangular storage areas (i.e., the optimal ratio between the length and width of the area), hence defining design benchmarks for the warehouse layout.

Later, the research article introduces by Bassan et al. (1980) studies the optimal layout for a rectangular warehouse depending on alternative shelf arrangements, labeled as the “homogeneous warehouse” and the “zoned warehouse”. Moreover, their optimization model provides guidance to determine the ideal location of docks, considering the internal handling costs. In a different fashion, the work of Ashayeri and Gelders (1985) provides a review of several warehouse design optimization models and introduces a general framework classification. More specifically, three different types of methods are identified:

1. Analytical models;
2. Simulation models;
3. Heuristic models.

According to the described categorization, the contribution of Ekren and Heragu (2010) adopts a simulation approach to identify the best configuration of racks for an autonomous vehicle storage and retrieval system (AVS/RS). Differently, the research of Pandit and Palekar (1993) proposes an analytical model for a rectangular warehouse with a multivehicle material handling system and describes the impact of layout on the response time of the handling system itself. Lastly, De Koster et al. (2017) perform a wide review of the top-cited papers in the warehouse design area since 1997, describing also their specific addressed subjects and main adopted methodologies.

However, it is possible to underline that none of the previously described contributions – either with the implementation of an analytical, a simulation, or a heuristic approach – has been able to move from the original assumption of Francis (1967) of a single point of input/output to the consideration of a lateral receive/ship dock for the rectangular warehouse. Indeed, it seems that only the book of Caron et al. (1997) has shortly discussed the abovementioned warehouse optimization problem, leaving decision-makers with ambiguity when required to determine the ideal design of modern distribution centers and logistics warehouses. For this reason, this technical note aims to fill the present void in the scientific background by proposing an analytical model for the layout optimization of a rectangular storage area that adopts a lateral receive/ship dock, keeping the assumption of a uniform access probability for the storage slots.

3. Demonstration of the Optimal Aspect Ratio

3.1. Notation

The following notation is introduced:

U	storage area width (latitudinal dimension)
V	storage area length (longitudinal dimension)
$A_{tot} = U \cdot V$	total storage area
x	variable describing the input/output point of the storage area
P_a	average travel length of the operators
$AR_o = U/V$	storage area optimal aspect ratio

3.2. Mathematical Demonstration

Figure 1 illustrates the rectangular warehouse design of a given area A_{tot}, where U and V represent the warehouse dimensions – respectively, the width and length – and I/O is the uniformly distributed dock on the latitudinal axis of the warehouse. The following basic assumptions are considered for this demonstration:

- Each point in the storage area has an equal probability of invoking a stow/pick operation, meaning that operator travels are uniformly distributed on the surface of the storage area (Francis, 1967);
- Each point along the input/output dock has an equal probability of generating a receive/ship operation, meaning that input/output operations are uniformly distributed along the latitudinal axis of the storage area.

With these assumptions, the average travel length of the operator to perform a stow or pick operation P_a depends on the input/output point of the storage area. However, given the uniformly distributed dock, the entry point is described with a random variable x with a continuous uniform distribution

$$x \sim U[a, b], \text{where } [a, b] = [0, U] \quad (1)$$

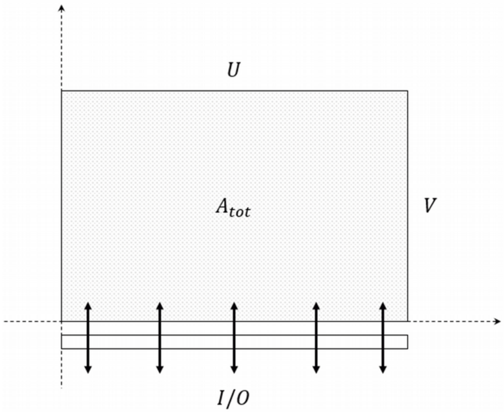

Figure 1. Storage area layout scheme (U = storage area width; V = storage area length; A_{tot} = total storage area; I/O = input/output of the warehouse)

hence, the random variable has the following probability density function (Montgomery & Runger, 2013)

$$f(x) = \frac{1}{b-a} = \frac{1}{U-0} = \frac{1}{U} \tag{2}$$

As a consequence, the average travel length P_a depends on x and can be written as $P_a(x)$.

To compute $P_a(x)$, the demonstration first considers the travel length along the longitudinal axis: since each point of the storage area has an equal probability of being accessed, in the best case the operator has to reach a point at the immediate entrance of the storage area, corresponding to the I/O point; thus the travel length is 0. In the worst case, the operator has to reach a point on the farthest side of the storage area; thus, the travel length is V. Thus, the average path on the longitudinal axis does not depend on the input/output point of the storage area, and is given as follows

$$P_{a,lon} = \frac{0+V}{2} = \frac{V}{2} \tag{3}$$

Differently, the travel length on the latitudinal axis directly depends on the input/output point of the warehouse, as illustrated in Figure 2. Depending on x, the warehouse is then divided into two different ordered rectangular layouts (Francis, 1967):

- $A_1 = x \cdot V$
- $A_2 = (U - x) \cdot V$

with $A_{tot} = A_1 + A_2$.

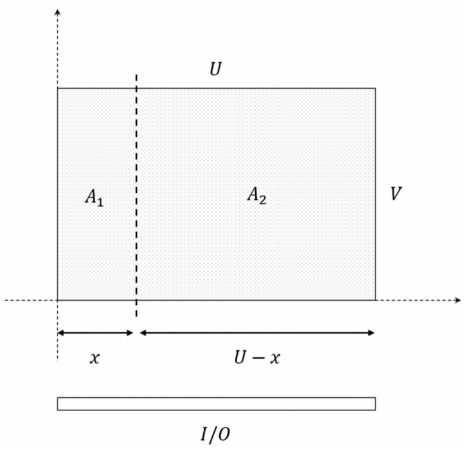

Figure 2. Warehouse layout and random variable definition

It now is possible to calculate the average latitudinal travel length of the operator in both areas: if the point to be reached by the operator on the latitudinal axis falls within A_1, the average latitudinal travel length of the operator is $\frac{x}{2}$; differently, if it falls within A_2, the average length is $\frac{U-x}{2}$. Hence, the overall average latitudinal travel length of the operator for the warehouse A_{tot} is computed by weighing the average paths for both areas A_1 and A_2, as follows

$$P_{a,lat}(x) = \frac{\frac{x}{2}\cdot A_1 + \frac{U-x}{2}\cdot A_2}{A_1 + A_2} = \frac{\frac{x}{2}\cdot A_1 + \frac{U-x}{2}\cdot A_2}{A_{tot}} \tag{4}$$

if $A_1 = x\,V$ and $A_2 = (U-x)V$ then

$$\begin{aligned} P_{a,lat}(x) = \; & \frac{\frac{x}{2}\cdot A_1 + \frac{U-x}{2}\cdot A_2}{A_{tot}} = \frac{\frac{x}{2}\cdot (xV) + \frac{U-x}{2}\cdot [(U-x)V]}{A_{tot}} \\ = \; & \frac{\frac{V}{2}\cdot x^2 + \frac{(U-x)^2}{2}\cdot V}{UV} = \frac{1}{2U}(2x^2 - 2Ux + U^2) \end{aligned} \tag{5}$$

At this point, given that the mean $E[g(x)]$ of a function $g(x)$, where x is a random variable, is computed through

$$E[g(x)] = \int [g(x)\cdot f(x)]dx \tag{6}$$

by applying (6) with $f(x) = \frac{1}{U}$ and $g(x) = P_{a,lat}(x)$ the expected value of the average latitudinal travel length for the operator is

$$\begin{aligned} E[P_{a,lat}(x)] = \; & \int_0^U \left[\left(\frac{1}{2U}(2x^2 - 2Ux + U^2)\right)\frac{1}{U}\right]dx \\ = \; & \left(\frac{1}{2U^2}\right)\int_0^U (2x^2 - 2Ux + U^2)dx \\ = \; & \left(\frac{1}{2U^2}\right)\left(\frac{2}{3}x^3 - Ux^2 + xU^2\right)\Big|_0^U = \frac{1}{2U^2}\cdot\frac{2}{3}U^3 = \frac{U}{3} \end{aligned} \tag{7}$$

hence, the overall average travel length for the operator results as follows

$$P_a(X) = E[P_{a,lat}(X)] + P_{a,lon} = \frac{U}{3} + \frac{V}{2} \quad (8) \tag{8}$$

also, the roundtrip length as a function of the known area A_{tot} yields

$$r = 2P_a(X) = \frac{2U}{3} + V = \frac{2U}{3} + \frac{A_{tot}}{U} = r(U) \tag{9}$$

the optimal aspect ratio AR_o of the storage area is hence found by minimizing the roundtrip length in (9)

$$\frac{\partial r(U)}{\partial U} = \frac{2}{3} - \frac{A_{tot}}{U^2} = 0 \Rightarrow U = \sqrt{\frac{3}{2}A_{tot}} \tag{10}$$

and given $A_{tot} = UV$ and $A_{tot} = \frac{2}{3}\,U^2$ it yields

$$V = \frac{A_{tot}}{U} = \frac{A_{tot}}{\sqrt{\frac{3}{2}A_{tot}}} = \sqrt{\frac{2}{3}A_{tot}} = \sqrt{\frac{2}{3} \cdot \frac{2}{3}U^2} = \frac{2}{3}U \tag{11}$$

therefore, obtaining the optimal aspect ratio of the storage area

$$AR_o = \frac{U}{V} = 1.5 \tag{12}$$

4. Conclusions

The research paper proposes the mathematical demonstration for obtaining the optimal ratio between the width (U) and length (L) of a rectangular storage area that adopts a lateral receive/ship dock, keeping the assumption of a uniform access probability for the storage slots. The procedure models the entry point of the storage area with a continuously distributed random variable and finds the optimal aspect ratio by minimizing the travel times of operators. The identified aspect ratio is U=1.5V.

This technical note is the first scientific contribution discussing the optimal aspect ratio of such storage area, which represents the common configuration of modern distribution centers and logistics warehouses. By adopting this aspect ratio, logistics managers, warehouse managers, and decision-makers would be able to determine the optimal dimensions for the design of a distribution center, with the aim of minimizing internal handling operations. Hence, the industrial applications of this scientific contribution's findings are straightforward and reported as follows:

1. Define the optimal design of a rectangular storage area with a lateral receive/ship dock;
2. Minimize the total travel times of operators and internal handling operations times;
3. Provide a decision-making tool to support logistics managers and warehouse managers in the design of distribution centers.

Moreover, further developments of this research work may consider other probability distributions for the storage area slots – which can be representative of specific industrial settings – with the aim of applying the proposed methodology to several different industrial backgrounds.

Acknowledgments

The authors would like to show their gratitude to Ludovico Lanni for his constructive comments in the discussion on the development of the demonstration.

Declaration of Conflicting Interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

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