OmniaScience

Journal of Industrial Engineering and Management
JIEM, 2024 – 17(3): 698-720 – Online ISSN: 2013-0953 – Print ISSN: 2013-8423
https://doi.org/10.3926/jiem.6792

Improving Production Sequencing in the Age of Industry 4.0: A Mathematical Launching Model Approach

Fatmir Azemi[1], Roberto Lujić[2], Goran Šimunović[2], Katica Šimunović[2], Faruk Ahmeti[1*]

[1]*University of Mitrovica "Isa Boletini", Faculty of Mechanical and Computer Engineering (Kosovo)*
[2]*University of Slavonski Brod, Mechanical Engineering Faculty in Slavonski Brod (Croatia)*

fatmir.azemi@umib.net, rljujic@unisb.hr, gsimunovic@unisb.hr, ksimunovic@unisb.hr
[*]*Corresponding author: faruk.ahmeti@umib.net*

Received: October 2023
Accepted: September 2024

Abstract:

Purpose: To enhance enterprise efficiency, this study examines the pivotal role of operations management in optimizing the use of materials, technology, equipment, and personnel, especially within the transformative framework of Industry 4.0.

Design/methodology/approach: This article investigates operational preparation in the context of Industry 4.0. Through questionnaires and interviews with stakeholders, problems related to operational preparation were identified. To address these problems, a launching model was developed for production systems. The model considers several parameters important for the production process, including business goals, customer needs, and environmental conditions that impact enterprise performance and profit.

Findings: The model determines the importance of jobs to be performed in a certain order to optimize production sequence. The proposed parameters are included in a mathematical-algorithmic launching model based on categorical levels related to the production process, such as profit, delivery times, processing times, total number of technological operations, product types, materials, required quality, product complexity, and resource use. The model was developed based on observations and investigations conducted in Kosovo's enterprises on operations research, and it has the potential to significantly improve production efficiency and profitability in the Industry 4.0 era.

Practical implications: The findings of this study have practical implications for operations management within the Industry 4.0 framework, proposing a launching model designed to optimize production processes and enhance efficiency.

Originality/value: The originality and value of this research lie in the development of a mathematical-algorithmic launching model that addresses operational preparation in the Industry 4.0 context, taking into account various crucial parameters.

Keywords: industry 4.0, launching model, operations management, efficiency, mathematical-algorithmic model

To cite this article:

Azemi, F., Lujić, R., Šimunović, G., Šimunović, K., & Ahmeti, F. (2024). Improving production sequencing in the age of Industry 4.0: A mathematical launching model approach. *Journal of Industrial Engineering and Management*, 17(3), 698-720. https://doi.org/10.3926/jiem.6792

1. Introduction

In today's globalized economy, manufacturing industries face intense competition to optimize their production processes and increase productivity. The fourth industrial revolution, also known as Industry 4.0, has brought about a new wave of technological advancements that can help industries to achieve these goals. The implementation of advanced technologies such as artificial intelligence, Internet of Things, and big data analytics can enable manufacturing enterprises to improve the utilization of materials, equipment, and personnel, leading to higher levels of efficiency and profitability.

To successfully implement Industry 4.0, research and development activities must be supported by appropriate industrial and policy decisions. The Industry 4.0 Working Group (Kagermann, Wahlster & Helbig, 2013; Lee, Bagheri, & Kao, 2015) emphasizes the necessity for action in eight key areas, including resource efficiency. In the context of production sequencing in the Era of Industry 4.0, this means optimizing resource use to enhance efficiency, reduce waste, and improve production flow. Effective production sequencing, driven by advanced technologies and data analytics, ensures that resources are allocated and utilized in the most efficient manner, aligning with the broader goals of Industry 4.0 for sustainable and intelligent manufacturing systems.

Despite the promise of technological advancements, many manufacturing enterprises, particularly those in Kosovo, continue to rely on traditional models for production planning and scheduling. Through extensive visits to enterprises in Kosovo over the past few years, it has become evident that a significant number of them still depend on manual paperwork for their scheduling operations. These manual processes, entrenched in traditional models, often prove inadequate in adapting to the complexities of modern manufacturing environments (Aranda-Jiménez, De-Pablos-Heredero, Campos-García, San-Martín & Cosculluela-Martínez, 2024). The observed reliance on manual methods highlights the challenges faced by manufacturing enterprises in integrating advanced technologies into their operations. Traditional models struggle to effectively integrate diverse data sources, respond to dynamic market demands, and optimize resource allocation. As a result, there is a pressing need for a new approach that leverages the power of technology to overcome these shortcomings and drive efficiency gains. Over the past few years, Industry 4.0 has emerged as a promising technological framework for integrating and enhancing manufacturing processes both within and between organizations (Xu, Xu & Li, 2018). Driven by recent advancements in ICT, Industry 4.0 provides a range of solutions to meet the increasing needs for digitalization in manufacturing industries. This framework's effectiveness is demonstrated by the growing number of companies globally that have explored the advantages of digitizing their horizontal and vertical chains, adopting Industry 4.0, and becoming leading digital enterprises in future complex industrial ecosystems.

Production sequencing, in particular, stands to gain substantially from these advancements in the Era of Industry 4.0 (Parente, Figueira, Amorim & Marques,, 2020). By harnessing real-time data, predictive analytics, and interconnected systems, production sequencing can be optimized for efficient resource allocation, minimized downtime, and improved responsiveness to market demands. The integration of these advanced technologies into production sequencing not only tackles current operational challenges but also sets the stage for more agile and resilient manufacturing systems.

Moreover, the adoption of advanced scheduling and sequencing software remains limited among manufacturing firms in Kosovo. Many companies persist in using manual methods or outdated software for critical tasks, thereby missing out on the efficiency gains offered by modern technologies. Our research seeks to bridge this gap by developing a comprehensive mathematical launching model tailored to the specific needs of manufacturing enterprises in Kosovo, particularly in the context of Industry 4.0.

By harnessing advanced mathematical algorithms and considering key parameters such as customer significance, resource utilization, and etc., our model aims to provide actionable insights for improving productivity and competitiveness. We are confident that by addressing the limitations of traditional models and embracing the potential of Industry 4.0 technologies, manufacturing companies in Kosovo can unlock new levels of efficiency and performance.

There are several ways to increase productivity, including reducing input data while maintaining the same level of output data, increasing output data with the same level of input data, or a combination of both. To achieve this, enterprises can eliminate waste, reduce material inputs, use improved technology, optimize resource utilization, and improve production design and management. They can also increase efficiency by minimizing maintenance downtime, improving the quality of goods, reducing inventory size, decreasing working capital requirements, and enhancing employee skills through training.

To implement these measures, enterprises require specific types of information. The necessary measures can be selected based on the information available and the objective of the investigation. Consequently, companies should perform an Industry 4.0 maturity assessment to clarify those issues and overcome uncertainty and potential problems (Ciravegna-Martins-da-Fonseca, Pereira Oliveira Ferreira & Busu, 2024). Maturity models describe the current scenarios of the organizations and offer improvement guidelines (Schumacher, Erol & Sihn, 2016). Furthermore, maturity models need to be easy to understand and apply by the companies, ensuring that they can effectively utilize the information gathered to select and implement the appropriate measures (Mittal, Khan, Romero & Wuest, 2018).

In the context of Production Sequencing in the Era of Industry 4.0, these assessments and models are particularly crucial. Production sequencing requires real-time data integration and advanced analytics to optimize resource allocation and enhance operational efficiency (Ivanov, Sokolov & Kaeschel, 2016). By leveraging Industry 4.0 maturity assessments, companies can better understand their current capabilities and identify the necessary steps to achieve more sophisticated and responsive production sequencing processes (Kolberg & Zühlke, 2015). These models provide a structured approach to integrating advanced technologies, ultimately supporting the transition towards more agile and resilient manufacturing systems.

Our proposed mathematical launching model is grounded in the principles of operations research and optimization theory. The model builds upon established concepts from scheduling theory, including job-shop scheduling, resource allocation, and queuing theory (Pinedo, 2016). The model integrates concepts from Industry 4.0, including real-time data processing and adaptive algorithms. By incorporating these advanced techniques, we aim to address the limitations of traditional scheduling models that often struggle to adapt to dynamic manufacturing environments (Xu et al., 2018). The theoretical framework is further supported by the application of heuristics and metaheuristics, which provide solutions for complex scheduling problems where exact methods may be computationally infeasible (Gonzalez, Kim & Choi, 2018).

The main purpose of this paper is to develop a mathematical launching model that can help manufacturing enterprises increase productivity by optimizing their operations sequence. The model will focus on the significant tasks of the production process, taking into consideration several parameters important for the production system.

In the manufacturing industry, one of the primary objectives of planning and scheduling is to reduce production time and costs by determining when to manufacture a product, which equipment and staff to use, and in what order. Production planning and scheduling are essential to increase the efficiency of the process and reduce the cost of production. To create a launching and scheduling model, mathematical models of processes, objective determination, and the relationship between resources and tasks must be defined under specific constraints.

The proposed model will determine the importance of each job in the production process in a certain order. Based on the research conducted through a questionnaire, the parameters having the greatest impact on the production system are proposed. These parameters include business goals, customer needs, and conditions from the surrounding environment that affect the enterprise, and can significantly increase its performance and profitability. The mathematical-algorithmic launching model will be developed based on several categorical levels related to the production process, such as profit, delivery times, processing times, the total number of technological operations, product types, materials, required quality, product complexity, and resource utilization.

The aim of the proposed model is to optimize the sequence of operations, increase profitability, reduce production time, and enhance the overall performance of the manufacturing system. The key inputs and outputs included in the model are detailed in Table 1.

The model will determine the sequence of jobs based on the significance value of each customer order, taking into consideration the most influential parameters for the enterprise. By implementing this model, manufacturing enterprises can improve their competitiveness and achieve higher levels of efficiency and productivity in the Industry 4.0 era.

Input Data				Output Data
Machine	Workforce	Product	Material	
Utilisation Type of machine Utilisation of machines	Number of employees that are needed for finishing jobs Experience	Processing time Due date Release date Customer significance Number of parts	Purchase time Material handling	Time model Launching the sequence model

Table 1. Inputs and outputs

2. Literature Review

In the manufacturing industry, the need to produce and supply customer-oriented products while maintaining profitability has become increasingly crucial due to rising competition in the open market. Product costs, often determined by competitors, necessitate strategies to reduce production and distribution costs to increase profits. To achieve this, efficient management and operations, along with the application of advanced production technologies, such as those offered by Industry 4.0, are imperative.

Among the crucial aspects of manufacturing efficiency, production planning and scheduling play a vital role. Traditional experience-based manual planning and scheduling often suffer from a lack of standardization, suboptimal optimization, long lead times, and high costs. To address these challenges, Computer-Aided Process Planning (CAPP) systems were developed to assist planners in their activities (Xu, 2011; Youssef & El-Hofy, 2008).

Industry 4.0 has revolutionized manufacturing by integrating advanced technologies and automated systems. Key elements of Industry 4.0 include the Internet of Things (IoT), artificial intelligence (AI), robotics, and big data analytics. These technologies enable real-time data collection, analysis, and optimization, resulting in increased productivity, reduced costs, and enhanced customer satisfaction (Kagermann et al., 2013).

Researchers have increasingly focused on leveraging Industry 4.0 technologies to improve production planning and scheduling. For instance, Zhang, Guo, Lv and Liu. (2018) proposed an intelligent production scheduling system that utilizes big data analysis and machine learning. The system employs a genetic algorithm to optimize the production plan and a deep reinforcement learning algorithm to optimize the scheduling plan. Results demonstrated that the proposed system outperformed traditional scheduling methods in terms of production efficiency and customer satisfaction.

Similarly, Gulivindala, Bahulalendruni, Chandrasekar, Ahmed, Abidi and Al-Ahmari (2023) developed an Industry 4.0-based production scheduling system that utilized a genetic algorithm to optimize the scheduling plan, accounting for various production constraints such as machine availability, tool availability, and maintenance downtime. The results indicated that the proposed system improved production efficiency and reduced production costs.

In addition to the aforementioned studies, various techniques and approaches have been formulated and developed over the past decades to address Job Shop Scheduling Problems (JSSP). These methods encompass Dispatch Rules (DP), Simulated Annealing (SA), Tabu Search (TS), Particle Swarm Optimization (PSO), Genetic Algorithm (GA), Artificial Immune System (AIS), Linear Programming (LP), Mixed Integer Linear Programming (MILP), among others. According to Schaefer (2011), JSSP is described as an NP-hard combinatorial optimization problem.

Applying Industry 4.0 technologies to the context of JSSP has also garnered attention. For example, Gao, Cao, Zhang, Chen, Han and Pan (2019) proposed a genetic algorithm-based method for solving JSSP, utilizing a parallel computing architecture to expedite the optimization process. The results indicated that the proposed method effectively and efficiently solved JSSP.

While significant progress has been made in using Industry 4.0 technologies to improve production planning and scheduling, there are still lack of Research on Manufacturing Sequences in the Context of Industry 4.0. There appears to be a gap in research addressing manufacturing sequences and this is highlighted especially in research gap in the Kosovo Manufacturing Industry. Manufacturing sequences refer to the optimal order in which manufacturing operations should be executed to achieve efficient production processes. Further exploration is needed to investigate how Industry 4.0 technologies can be leveraged to optimize manufacturing sequences, considering factors such as resource utilization, production constraints, and customer requirements.

In the Era of Industry 4.0, Production Sequencing has undergone significant transformation due to advancements in digital technologies and automation. Industry 4.0 is characterized by the integration of cyber-physical systems, the Internet of Things (IoT), and big data analytics into manufacturing processes, which has profound implications for production scheduling and sequencing (Kagermann et al., 2013; Lee et al., 2015).

The traditional methods of production sequencing, which primarily focused on optimizing the order of operations to minimize downtime and maximize efficiency, are now being supplemented with advanced algorithms and real-time data processing capabilities (Wang, Yan & Zhang, 2016). For instance, the use of machine learning algorithms has enabled more dynamic and adaptive scheduling systems that can respond to real-time changes in production conditions (Choi, Wallace & Wang, 2018).

Moreover, the implementation of IoT technologies allows for greater visibility and control over the production process, enabling more precise and timely adjustments to sequencing plans (Brettel,Friederichsen, Keller & Rosenberg, 2014). These technological advancements not only enhance operational efficiency but also contribute to the overall flexibility and responsiveness of manufacturing systems (Hazen, Boone, Ezell & Jones-Farmer, 2016).

3. Problem Description and Assumption

In most enterprises in Kosovo, the manufacturing processes are traditionally operated based on past experiences (as determined through interviews and questionnaires). This is a common practice in many developing countries, as indicated by the literature review.

The implementation of launching sequences has been analysed using various methods in recent years. Although certain models and priority rules have been identified as useful in several reports, these methods have limitations that are primarily related to business objectives, customer demands, operational characteristics, and can vary from one enterprise to another.

A set of customer orders, $\{1, ..., n\}$, has been identified as jobs. For example, the first customer order is referred to as *Job1*, the second customer order as *Job2*, and so on. Each order, n, consists of a set of products, p, with associated operations, o. The problem is to determine the launching sequence of jobs based on the significance value of each customer order.

Therefore, the model will be developed based on parameters related to business objectives, customer needs, and environmental conditions within the enterprise.

These parameters will be divided into four levels, each with its own set of influence parameters that are deemed most important for determining the launching sequence.

The launching sequence of jobs (*LSJ*) can be summarized using the following expression (Expression 1) across the aforementioned levels:

$$LSJ = f\{COS, PS, MS, TPS\} \tag{1}$$

Where:

LSJ – Launching Sequence of Jobs
COS – Customer Order Significance
PS – Product Significance

MS – Material Significance
TPS – Technological Processes Significance.

The task at hand is to fulfil a set of n customer orders, each consisting of p products. Each product requires o operations to be completed and is divided into l parts to be processed on m machines. Several parameters are defined for each product, including:

- Setup Time: The time required for setting up the machines and resources before starting the production of a specific product.
- Processing Time per Piece: The duration needed to process a single piece of a product on a machine.
- Number of Pieces: The total quantity of pieces required to complete the production of a particular product.
- Delivery Date of Product: The deadline or expected delivery date for each product as per the customer's requirements.
- Price of Product: The cost or price associated with producing and delivering the product.
- Availability of Machines: The availability and capacity of machines, which is typically expressed in units of time.

These parameters play a crucial role in determining the production schedule, optimizing resource allocation, and meeting customer demands within the specified time frame. By considering these parameters and their values for each product, efficient production planning and scheduling can be achieved.

3.1. Model Assumption and Parameters

The launching model for determining the sequence of operations in the production process is underpinned by a set of assumptions and limitations, which guide its framework and application. These assumptions are as follows:

1. No Arrival of New Jobs: The analysis assumes that no new jobs arrive during the sequencing process. It focuses on determining the optimal sequence for the existing set of jobs.
2. Sequence of Operations: The model assumes that the sequence of operations must be followed as specified by the technological procedure. It does not consider the possibility of altering the predefined sequence.
3. Availability of Jobs: It is assumed that all jobs are known and ready to start processing before the period under consideration begins. The model does not account for new jobs entering the system during the sequencing process.
4. Interoperative Losses: The model considers interoperative losses such as setup time and transport time in the initial sequencing part. However, these losses are not taken into account during the scheduling process. It is assumed that each subsequent operation starts after the completion of the previous one.
5. Capacity-Based Operations: Operations are performed based on the availability of capacity, specifically the availability of machines. The model takes into account the capacity constraints when determining the launching sequence.
6. Interruption of Operations: At a given time interval, when new products need to enter the production process, all operations, except those currently in progress on the machines, will be interrupted. This interruption allows for the inclusion of new jobs in the sequencing calculations.
7. Completion of Started Procedures: Operations that are already in progress on the machines will be observed until the completion of their started procedures. Any remaining unfinished operations on the product, along with the newly arrived jobs, will be considered in the calculation of the launching sequences.

These assumptions define the scope and conditions under which the launching model operates, providing a structured framework for sequencing and scheduling processes.

In addition to the assumptions, limitations of the launching model include:

1. During the analysis, new jobs are not arriving,
2. The sequence of operations must be followed as specified by the technological procedure,
3. All jobs are known and are ready to start processing before the period under consideration begins,
4. Interoperative losses (setup time, transport time...) will be considered in the first part of sequencing but they will not be taken into consideration during the scheduling process (it is calculated that each subsequent operation begins after the completion of the previous one),
5. The operations are performed according to the availability of capacity (machines),
6. At a given time, interval (when and if the new products have to enter the production process), all operations (except those currently in the production process on the machine), will be interrupted,
7. The operation that is on the machine will be observed until the end of the started procedure of operation. All remaining unfinished operations on the product (whose production was started) enter into the calculation of launching sequences together with the newly arrived jobs.

In the proposed model, the sequence of customer orders is determined with the objectives of meeting due dates and minimizing production costs. The model takes into account the presence of different jobs in the production system, where technological operations may have capacity requirements that lead to "waiting" at workstations. The significance of each customer order is determined as a basis for the scheduling process, considering the following important parameters:

Customer order: Customer relevance, Quantity of different products in the order, Number of possible batches, the complexity of providing resources per customer order, the financial contribution of own funds in the realization of the customer order, delivery date (due date), profit per customer order.

Product: Product complexity, Product type, The complexity of quality.

Material: Availability of readiness of necessary materials, Complexity factor of securing other resources (documentation, tools, safety equipment, protective products, …).

The technological process: Number of technological operations, The possibility of alternative technological processes, The factor of capacity availability.

By considering these parameters, the model aims to calculate the production significance of each customer order, which forms the basis for the subsequent scheduling process. The significance values obtained from the model can guide decision-making in determining the optimal sequence of customer orders to meet due dates and minimize production costs in the manufacturing system.

4. Methodology

The methodology for this research involves the development of a mathematical model based on parameters related to business objectives, customer needs, and environmental conditions within the enterprise. The focus is on identifying the parameters that have the most influence on the production process, which are then incorporated into the launching model. The model aims to determine the significance value of tasks (jobs) either in the production phase or to be produced.

In order to gain a comprehensive understanding of the operational landscape within manufacturing enterprises in Kosovo, our study involved extensive on-site visits, interviews, and observations. These visits provided valuable insights into the industry-specific challenges faced by manufacturing enterprises, including the low level of software application and the absence of planning and scheduling models. Through structured interviews with industry professionals and stakeholders, we collected data on the operational readiness of manufacturing enterprises in adopting Industry 4.0 practices.

To gather data on the influential parameters, a survey was conducted among the 22 most successful enterprises in Kosovo. Visits were made to these enterprises, and a questionnaire was distributed to gather information about the parameters that impact the determination of task significance. A total of 35 questionnaires were distributed, and 27 valid responses were received, resulting in a survey response rate of 77.14%.

The questionnaire was designed based on a thorough review of existing literature, which encompasses theories and practices relevant to production sequencing and Industry 4.0 principles. This literature review guided the identification of key parameters that influence the determination of job significance within manufacturing enterprises. By grounding our questionnaire in established theories and empirical evidence, we aimed to ensure its validity and relevance to the research objectives.

Moreover, the survey questionnaire underwent a rigorous validation process to ensure its effectiveness in capturing pertinent data. This involved consulting with domain experts and practitioners in the field of manufacturing operations management to review and refine the questionnaire items. Their insights and feedback were instrumental in enhancing the questionnaire's comprehensiveness and clarity.

Additionally, the survey methodology incorporated established principles of survey design to optimize data collection and ensure the reliability of responses. This included using clear language, providing exhaustive response options, and pre-testing the questionnaire with a small sample to identify and address any potential ambiguities or issues.

Overall, the survey questionnaire was meticulously developed based on theoretical foundations, empirical evidence, and input from industry experts to ensure its alignment with best practices in the field. By adhering to these rigorous methodological standards, we aimed to obtain high-quality data that could inform the development of the launching model and contribute to advancements in production planning and scheduling processes within the Industry 4.0 context.

The collected data and insights from the questionnaire responses were used to inform the development of the launching model. Both heuristic approaches and modelling techniques were employed in creating the model. The aim is to provide the companies with a cutting-edge tool that can assist them in their daily planning and scheduling activities, while embracing the transformative capabilities of Industry 4.0.

Through the integration of survey findings with a mathematical model inspered with Industry 4.0 principles, this research aims to provide practical solutions and insights for enhancing production planning and scheduling processes within the context of Industry 4.0 in surveyed enterprises in Kosovo.

5. The Mathematical Model

The launching sequence of jobs is determined based on the significance of each customer order. This model calculates the significance value for each job that enters the production process, and then arranges a sequence for launching products accordingly. In this scenario, each customer order represents a single job, consisting of a set of products with multiple operations. The number of machines is denoted by *'m',* and the number of jobs by *'n'.* Typically, the subscript *'i'* denotes a job, while the subscript *'j'* refers to a machine. If a job requires multiple processing steps or operations, the pair (i, j) represents the specific step or operation of job *'i'* on machine *'j'.*

The proposed model for launching sequences is as follows:

Indices

$i = 1,2,3,\dots, n; i$ – customer order (jobs),
$p = 1,2,3, \dots pp, p$ – number of products,
$o = 1, 2, 3, \dots, oo; o$ – number of operations,
$l = 1, 2, 3, \dots, ll; l$ – number of parts.

Parameters

x_1 to x_7 – weighting factors determined through a digital logic model.

Nomenclature

LSJi – Launching Sequence of the i^{th} customer order (job),
COSi – Customer Order Significance of the i^{th} job,
PSi – Product Significance of the i^{th} job,
MSipl – Material Significance of the i^{th} job, the p^{th} product and the l^{th} part,
TPSiplo – Technological Processes Significance of the i^{th} job, the p^{th} product, the l^{th} part and o^{th} operation.
CRi – Customer Relevance factor,
QDPi – Quantity of Different Products in the customer order,
NBRi – Number of Batches Required factor,
CPRi – The Complexity of Providing Resources per customer order,
FCi – Financial Contribution factor,
DDi – Delivery Date (Due Date) factor,
EPi – Expected Profit factor for the i^{th} customer order,
Qopti – Optimal Batch quantity [piece],

Qil – Demands [piece],
OCil – Order Cost [€],
HCil – Holding Costs per piece [€/piece].
MTik – Manufacturing (Processing) Time on kth capacity for i^{th} order,
SuTplo – Setup Time of the p^{th} product, l^{th} part, o^{th} operation on the i^{th} customer order,
Qpi – Number of the p^{th} product on the i^{th} customer order,
LTplo – Loading Time of the p^{th} product, o^{th} operation, l^{th} part on i^{th} customer order,
UTplo – Unloading Time of the p^{th} product, o^{th} operation, l^{th} part on i^{th} customer order,
OTplo – Operation Time of the p^{th} product, o^{th} operation, l^{th} part on i^{th} customer order,
TTplo – Total Transport Time between o^{th} and $(o-1)^{th}$ operations of the p^{th} product, l^{th} part on the i^{th} customer order,
DTOplo – Delays Time between Operations – Machine idle Time.
DDi – Due Date, the deadline for delivery of the i^{th} customer order
LD – Launching Date
Zk – Busyness of k^{th} capacity in the planning period (from launching time)
FCk – Capacity Factor for the i^{th} customer order,
PCip – The Product Complexity factor
PTip – The Product Type factor
RQip – The Required Quality factor.

Binary decision variables

$$FC_k=\begin{cases} 1, & \text{if } k^{th} \text{ capacity is used on the } i^{th} \text{ customer order} \\ 0, & \text{if } k^{th} \text{ capacity is not used on the } i^{th} \text{ customer order} \end{cases}$$

5.1. Proposed Mathematical Model

$$LSJ_i=COS_i+PS_i+\sum_{p=1}^{pp}\sum_{l=1}^{ll} MS_{ipl}+\sum_{p=1}^{pp}\sum_{l=1}^{ll}\sum_{o=1}^{oo} TPS_{iplo} \tag{2}$$

$$COS_i=x_1\cdot CR_i+x_2\cdot QDP_i+x_3\cdot NBR_i+x_4\cdot CPR_i+x_5\cdot FC_i+x_6\cdot DD_i+x_7\cdot EP_i \tag{3}$$

$$COS_i = 0.190 \cdot CR_i + 0.095 \cdot QDP_i + 0.047 \cdot NBR_i + 0.095 \cdot CPR_i + \\ + 0.095 \cdot FC_i + 0.238 \cdot DD_i + 0.238 \cdot EP_i \tag{4}$$

$$Qopt_i = \sqrt{\frac{2 \cdot Q_{il} \cdot OC_{il}}{HC_{il}}} \ [piece] \tag{5}$$

$$MT_{ik} = \sum_{p=1}^{pp} \sum_{l=1}^{ll} \sum_{o=1}^{oo} \left(SuT_{plo} + Q_p \cdot \left(LT_{plo} + UT_{plo} \right) \right) \\ + \sum_{p=1}^{pp} \sum_{l=1}^{ll} \sum_{o=1}^{oo} (TT_{plo} \\ + DTO_{plo}) \tag{6}$$

$$CA_i = \frac{\sum_k^{kk} MT_{ik}}{\left(\left(\frac{\sum_i^n DD_i}{n} - LD \right) \cdot \sum_k^{kk} FC_k \right) - Z_k} \tag{7}$$

$$PS_{ip} = \sum_{p=1}^{pp} (PC_{ip} + PT_{ip} + RQ_{ip}) \tag{8}$$

$$MS_{ipl} = \sum_{p=1}^{pp} \sum_{l=1}^{ll} (ARNM_{ipl} + CSOR_{ipl}) \tag{9}$$

$$TPS_{iplo} = \sum_{p=1}^{pp} \sum_{l=1}^{ll} \sum_{o=1}^{oo} \left(NTO_{iplo} + ATP_{iplo} + CA_{iplo} \right) \tag{10}$$

$$LSJ_i = COS_i + PS_i + \sum_{p=1}^{pp} \sum_{l=1}^{ll} MS_{ipl} + \sum_{p=1}^{pp} \sum_{l=1}^{ll} \sum_{o=1}^{oo} TPS_{iplo} \tag{11}$$

$$LSJ_i = 0.190 \cdot CR_i + 0.095 \cdot QDP_i + 0.047 \cdot NBR_i + 0.095 \cdot CPR_i + \\ + 0.095 \cdot FC_i + 0.238 \cdot DD_i + 0.238 \cdot EP_i + PC_i + PT_i + RQ_i + \\ + \sum_{p=1}^{pp} \sum_{l=1}^{ll} (ARNM_{ipl} + CSOR_{ipl}) + \sum_{p=1}^{pp} \sum_{l=1}^{ll} \sum_{o=1}^{oo} (NTO_{iplo} + ATP_{iplo} + CA_{iplo}) \tag{12}$$

The relation (2) presents the overall expression of the launching sequence for job production, taking into account the operations of products on each machine. The Customer Order Significance value (*COSi*) for the i^{th} customer order is calculated using the general formula (3). In order to assess the impact of each individual parameter, a combination of expert knowledge, questionnaire data, and a digital-logical model was utilized. Through this process, the weight values for each influential parameter were determined (Table 2).

Furthermore, to mitigate the undue influence of less significant factors, the range of parameter values was normalized within a range of 1 to 5. As mentioned earlier, the Customer Order Significance value ($COSi$) for the i^{th} customer order is determined using expression (4). The calculation of the Economic Batch Quantity (Qopt) can be derived from expression (5). The total manufacturing time is represented by expression (6). In order to determine if the capacity for the i^{th} customer order meets the conditions for timely delivery to the customer, expression (7) is applied, and the corresponding factor is selected based on the calculated values of CA_i and DD_i for Customer Order Significance.

	CR_i	QDP_i	NBR_i	CPR_i	FC_i	DD_i	EP_i		Influence of a single factor
CR_i		1	1	1	1	0	0	4	=4/21= 0.190
QDP_i	0		1	0	1	0	0	2	=2/21=0.095
NBR_i	0	0		1	0	0	0	1	=1/21=0.047
CPR_i	0	1	0		1	0	0	2	=2/21=0.095
FC_i	0	0	1	0		1	0	2	=2/21=0.095
DD_i	1	1	1	1	0		1	5	=5/21=0.238
EP_i	1	1	1	1	1	0		5	=5/21=0.238
								21	

Table 2. Digital-logical model

The expression (8) determined Product Significance value PS_i of the i^{th} customer order. Similarly, the Material Significance value MS_i of the i^{th} customer order is determined by expression (9). The expression (10) determined the significance value of the complexity of the technological process TPS_{iplo}. Finally, the expression (11) and (12) presents the mathematical empirical formula of launching model for the job significance value and launching sequence, after all the parameters have been included in expression 2.

5.2. Determination of Factors

Customer satisfaction and relevance are key factors for the success of enterprises worldwide. As customer satisfaction serves as an indicator for the future of the enterprise (Russell & Millar, 2014), it is essential for the enterprise to efficiently develop and manufacture products that meet customer preferences.

Customer relevance, which is determined based on experience and expert knowledge, can be influenced by factors such as the frequency of orders, profit generated in previous periods, fulfillment of payment obligations, and expected profit per customer order (typically assessed by the sales and accounting department).

Based on the analysis of data from the questionnaire, the customer importance factor is assigned values as shown in Table 3.

Customer relevance factor	Value
Customers who are well-known for the enterprise	4.9
New customers who are expected to have a long-term cooperation	3.5
Medium-sized customers per payment	2.9
New customers who are not well-known for the enterprise	2.5
Unreliable customers	1.0

Table 3. Customer relevance factor

A customer of very special importance, who is well-known to the enterprise and has consistently completed transactions and payments on time with good profitability, is assigned a value of 4.9. A new customer with potential for long-term cooperation is assigned a value of 3.5. Medium-sized customers per payment receive a value of 2.9. A value of 2.5 is given to a new customer who is not well-known, and a value of 1 is assigned to less significant and unreliable customers.

The quantity of different products in a customer order has a direct impact on the complexity of both the customer order and the production process. This factor is determined by the production preparation department and is assigned values as presented in Table 4.

Quantitatively	Qualitatively
1	One type
3	Small
5	Large

Table 4. Significance value of Quantity of Different Products

According to expert knowledge from the production preparation department, the following values are assigned based on the quantity of different products in the customer order:

- A value of 1 is assigned when there is only one type of product in the customer order.
- A value of 3 is assigned when there is a medium quantity of different products in the customer order.
- A value of 5 is assigned when there is a high quantity of different products in the customer order.

At the time of launching, if there is a sufficient amount of material in stock to fulfil the entire customer order, one batch is launched into production. However, if there is a shortage of materials, the optimal batch size needs to be calculated.

The lack of material stocks can lead to production delays and extensions of delivery dates, resulting in direct and indirect losses for the enterprise. On the other hand, maintaining large material stocks ensures smooth production but can strain the enterprise's liquidity.

To determine the optimal batch size and achieve the minimum total cost per unit of product, the following factors should be considered (refer to Fig.2) (Majdandžić, Lujić, Simunović & Majdandžić, 2001):

1. Inventory Costs: These costs encompass various expenses associated with maintaining material stocks, including:
   - Interest on working capital invested in material stocks.
   - Costs of storage space required to store the materials.
   - Costs of inventory management and handling within the warehouse.
   - Costs of maintaining and operating the storage facilities.
2. Order Costs: These costs are incurred when placing orders for materials and include:
   - Transportation costs associated with shipping materials to the production facility.
   - Customs costs, if applicable.
   - Insurance costs for the transportation and storage of materials.
   - Operating costs related to order processing and preparation.
3. Material and Manufacturing Costs: These costs are assumed to be constant per unit of product and include expenses related to raw materials, production processes, and labor.

Inventory costs are directly proportional to the quantity of materials ordered, meaning that higher quantities result in higher inventory costs. On the other hand, order costs remain constant regardless of the quantity of materials ordered. However, the cost per unit of production decreases as the quantity of materials ordered increases.

While the optimal order sizes may not be explicitly considered in the budget, it is important not to overlook the costs associated with material shortages. Insufficient material stocks can lead to situations where the enterprise runs "out of stock." In such cases, additional costs may arise due to urgent orders, overtime work, expedited transportation to clients, contract terminations, delayed sales (resulting in penalty costs for missing deadlines), or the need to replace originally selected materials.

The total cost curve per unit of product, illustrated in Figure 2, exhibits a concave shape. This indicates that there exists a minimum value of total costs, corresponding to the optimal quantity of ordered materials. By identifying this optimal point, the enterprise can minimize its overall costs and achieve efficient resource allocation.

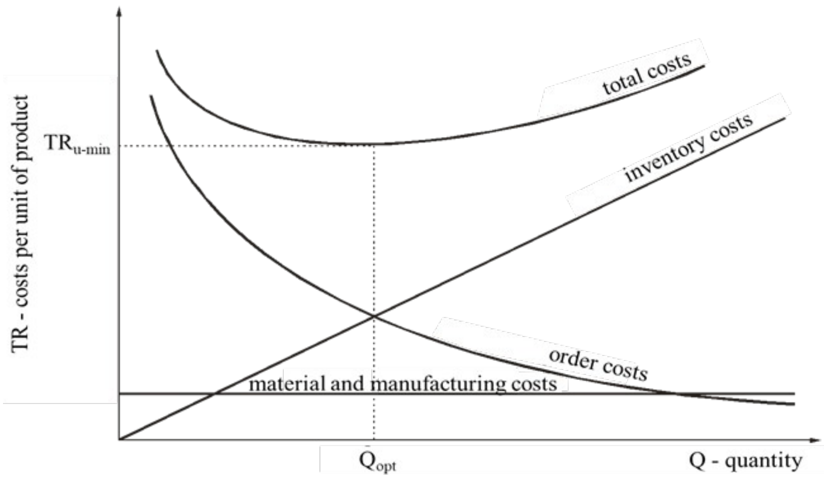

Figure 2. Determining the optimal order size (Majdandžić et al., 2001)

The Economic Batch Quantity (*Qopt*) can be calculated using expression (4.10). It represents the optimal quantity of materials to be ordered in each batch, considering the trade-off between inventory costs and order costs.

The number of batches required factor for the i^{th} customer order can be determined based on the calculated *Qopt* value. The values for this factor are presented in Table 5. The specific value depends on the quantity of materials required for the customer order and is determined to optimize the overall cost efficiency of the production process.

Quantitatively	Qualitatively
1	Launch of one series
3	Launch of a small number of series
5	Launch of a huge number of series

Table 5. The Number of Batches Required factor

The complexity of a customer order is influenced by the various ways resources are utilized in its realization. This includes the involvement of resources from the own enterprise, cooperation with external partners, and the number of participants involved in the execution of the customer order.

Based on the information provided by the manufacturing department and their expertise, the complexity factor of the i^{th} customer order can be determined. This factor, which reflects the complexity of providing resources per customer order, is assigned the following values:

- A value of 1 is assigned to customer orders that are completely handled by resources from the own enterprise.
- A value of 2 is assigned when some operations are outsourced to external cooperators.
- A value of 3 is assigned when some parts of the customer order are outsourced to external cooperators.

- A value of 4 is assigned when some products are outsourced to external cooperators.
- A value of 5 is assigned when the entire customer order is outsourced to external cooperators.

Table 6 presents the assigned values for the Complexity of Providing Resources per customer order (CPR_i) based on the level of complexity determined by the production preparation department.

Quantitatively	Qualitatively
1	Own enterprise resources
2	Some operations sent to cooperators
3	Some parts sent to cooperators
4	Some products sent to cooperators
5	Whole customer order sent to cooperators

Table 6. The Complexity of Providing Resources factor

The financial contribution per customer order (FC_i) represents the financial impact of own funds in the realization of the customer order and is considered a crucial factor. The determination of this factor is carried out in the sales and accounting departments, taking into account expert knowledge and information gathered from questionnaires. The financial contribution per customer order can take on various values, as shown in Table 7, reflecting different levels of financial impact and significance.

Quantitatively	Qualitatively
1	None
2,3	Small
3,6	Medium
4,9	Own resources

Table 7. Financial Contribution factor

If the financial contributions of own funds are not involved in the realization of the customer order, the factor FC_i takes a value of 1. On the other hand, if a small financial contribution of own funds has taken place in the realization of the order, the factor FC_i is assigned a value of 2.3. For cases where a medium financial contribution of own funds is required for the realization of the order, the factor FC_i takes a value of 3.6. Finally, when the realization of the customer order is entirely dependent on the contribution of own financial funds, the factor FC_i is assigned a value of 4.9. These values reflect the level of financial impact and significance of own funds in the execution of the customer order.

The delivery date is a crucial aspect in satisfying customer expectations and meeting their needs. Customers desire products to be delivered at the right time and in the right quantity. As stated by Kumar and Kumar (Kumar & Kumar, 2004), delivering the required function involves ensuring that the product meets quality, reliability, and maintainability requirements, is delivered from a reliable source, provides adequate pre- and post-sales service, and is offered at the right price.

The significance of the production sequence for a particular product increases as the remaining time to its deadline decreases. Equation (7) determines whether the capacity for the i^{th} customer order fulfils the conditions to deliver the order on time. The factor is determined based on the calculated values of CA_i, and the value of DD_i for the Customer Order Significance is selected accordingly. Table 8 provides the values for different scenarios:

- If the value of CA_i is less than 1, it indicates that the task is completed before the deadline.
- If CA_i is equal to 1, it means the task is on time.
- If CA_i is greater than 1, it signifies a delay.

Table 8 outlines the factors associated with the calculated values of CA_i and the corresponding significance values for meeting the delivery deadline.

CAn	DDn	Significance
<1	1	Before
=1	3	On-time
>1	5	Delays

Table 8. Deadline Delivery factor

The primary objective of any enterprise is to generate profit. Profitability is influenced by the production costs associated with each product. If the production costs are lower than the sales price, the enterprise will achieve financial gain. Conversely, if the production costs exceed the sales price, the enterprise will experience a financial loss.

The Expected Profit factor (EP_i) for the i^{th} customer order, as determined through questionnaires and interviews conducted with the enterprise management, is presented in Table 9. This factor quantifies the anticipated profitability associated with each customer order.

Profit %	Significance of EPi
<1	1
[1-3)	1,8
[3-5)	2,6
[5-7)	3,4
[7-8)	4,2
>8	5

Table 9. Expected Profit factor

Researchers have approached the measurement and definition of product complexity in various ways, leading to a lack of consensus on a universally accepted approach (Orfi, Terpenny & Sahin-Sariisik, 2011). One common perspective is to describe product complexity in terms of diversity, with some researchers considering diversity as synonymous with product complexity. Diversity refers to the degree of differences among components in terms of size, shape, material, number of parts, and other characteristics (Berger, Draganska & Simonson, 2007). Components with diverse attributes may require distinct quality assurance, manufacturing processes, and information management strategies.

Manufacturers often aim to reduce complexity by employing parts commonality and product platform sharing techniques, thereby minimizing the number and variety of parts used (Robertson & Ulrich, 1998). Novak and Eppinger (2001) propose three elements for characterizing product complexity: the number of components required to specify and produce a product, the level of interactions between these components, and the degree of product novelty. Additionally, the complexity of the manufacturing environment increases with the number of unique parts needed to build the final product (Ramdas & Sawhney, 2001).

Based on the aforementioned literature and other sources (Tomiyama, D'Amelio, Urbanic & Eimaraghy, 2007; Ulrich & Eppinger, 2012; Zhu, Hu, Koren & Marin, 2008; Novak & Eppinger, 2001; Bozarth, Warsing, Flynn & Flynn, 2009), key factors contributing to product complexity include size variety, material variety, design variety, product variety, process variety, number of parts or components, interdependencies between parts or components, level of product innovation, complexity of product structure, and more.

Information about the factors influencing product complexity can be obtained from the received orders, and the following elements should be considered:

- simple products with no subordinate elements,
- elements in structure,
- simple structural components,
- complex structural components (multilevel), and
- combined components.

Table 10 provides the values for the product complexity factor based on expert knowledge and considering all the mentioned elements.

Quantitatively	Qualitatively
1	Simple products
2	Elements in the structure
3	Simple structural component
4	Complex structural component
5	Combined component

Table 10. Product Complexity factor

The product type factor takes into account the change in the type of orders received. When a new order involves a product that is different from previous orders, it requires more attention and time for developing a new production program. This factor reflects the importance and complexity associated with such orders. Table 11 provides the values for the product type factor, which are based on the experience and expert knowledge gathered from operational preparation.

Quantitatively	Qualitatively
1	Product from existing production program
3	The existing product with changing requirements
5	New product

Table 11. The Product Type factor

The factor of the required quality for the i^{th} customer order is determined based on the data analyzed from the questionnaire. Table 12 provides the values and significance of the Required Quality factor (RQ_i). The correspondents' answers reflect the level of requirements for accuracy in measures and shapes. The table categorizes the values into high requirements, average accuracy requirements, and low requirements for measures and shapes. The assigned values represent the significance of the Required Quality factor for each category.

	RQi
Production parts of very high requirements of the accuracy of dimensions	5
Production parts of high requirements of the accuracy of dimensions	4
Production parts of medium requirements of the accuracy of dimensions and forms	3
Production parts of low requirements of the accuracy of dimensions and forms	2
Production parts of very low requirements of the accuracy of dimensions and forms	1

Table 12. The Required Quality factor

A value of 5 is assigned for production parts with very high requirements for the accuracy of dimensions and forms. A value of 4 is assigned for production parts with high requirements for the accuracy of dimensions and forms. A value of 3 is assigned for production parts with medium requirements for the accuracy of dimensions and forms. A value of 2 is assigned for production parts with low requirements for the accuracy of dimensions and forms. Lastly, a value of 1 is assigned for production parts with very low requirements for the accuracy of dimensions and forms.

The Availability and Readiness of Necessary Materials factor is assigned during production preparation. The supply chain within an organization controls and manages the flow of materials throughout the company, starting from the phase of purchasing raw materials to the phase of shipping final products to customers. To prevent delays in production and prioritize activities, it is important to determine the quantities and timing of components and materials, as well as their availability and the necessary actions to meet due dates and delivery deadlines. Materials Management is a function that integrates various aspects of materials management in an industrial enterprise, including purchasing, inventory control, storage, material handling, standardization, and more. Its main objective is to ensure a continuous supply of raw materials. Providing materials for production on time enhances the efficiency of production systems. The complexity factor related to the provision of resources (materials) is considered during production preparation. This factor aims to address potential failures in material supply for production. When determining this factor, factors such as delivery time, price, delivery quantity, and deferred payment need to be taken into account.

The table below presents values for some of the factors assigned based on experience and expertise. By combining the factors of time, price, quantity, and payment, as well as analysing data from Questionnaire 3 (Question 3.2), recommendations have been made for the availability and readiness of necessary materials in Table 13.

Delivery time		Delivery quantity		Price factor		Deferred payment	
1	Long time	1	Some parts	1	Low price	1	Yes
2	Medium	2	Half of the parts	2	Medium price	2	No
3	Short	3	All parts	3	High price		
4	Emergency						

Table 13. Combination of factors

Based on the analysed data (Questionnaire 3, Question 3.2), the availability and readiness of necessary materials factor takes the values as shown in Table 14. This factor is assigned in the production preparation process to ensure that the required materials are available and ready for production.

	ARNMi
Smaller quantities of various special requirements material	4.9
Medium quantities of various materials from different suppliers	4.4
Larger quantities of materials (supplier's standard deliveries)	3.2
The material provided (basic materials)	2.2
The material provided (auxiliary and consumables material)	1

Table 14. The Availability and Readiness of Necessary Materials factor

The Complexity of Securing Other Resources factor is assigned in the production preparation and takes into consideration accompanying resources, such as materials and equipment necessary in addition to the main resources. This group of resources includes documentation, tools and instruments, safety equipment, protective products, and so on. The complexity in securing these additional resources is not directly linked to the production process itself, but it supports the smooth flow of the process. Based on experience and expert knowledge, the CSORi factor takes the values as shown in Table 15.

As it seen from table 15, in the case of securing other resources from own resources, the factor takes a value of 1. In the case of limited and needed additional resources, the factor takes a value of 3. And in the case when securing of other resources will be fulfilled from external resources, the factor takes a value 5.

Value	Significance
1	Insignificant
3	Medium
5	High

Table 15. The Complexity of Securing Other Resources factor

The factor of the Number of Technological Operations (NTO_{iplo}) is assigned by the preparation technology department, and its values can be expressed based on expert knowledge as shown in Table 16.

Quantitatively	Qualitatively
1	Simple process plan
3	Medium process plan
5	Complex process plan

Table 16. The Number of Technological Operation factor

The ATP_{iplo} factor represents the opportunity to change possible technological procedures with alternatives if necessary. The data for this factor are collected through Questionnaire 3 (Question 3.4) and presented in Table 17.

	ATPiplo
Combined processes	5
Sending the whole part to cooperator	3.6
Sending part of some technological operations to cooperator	1.9

Table 17. The Alternative Technological Processes factor

A value of 5 is assigned to customer orders with the possibility of combined processes. A value of 3.6 is assigned to customer orders with the possibility of sending the whole part to a cooperator. A value of 1.9 is assigned to customer orders with the possibility of sending part of some technological operations to a cooperator.

Based on the capacity calculation described by expression (7), the factor of available capacity for the i^{th} customer order is determined. This factor is assigned in the preparation for production, specifically within the planning department. Table 18 gives values assigned to the capacity available factor.

Quantitatively	Qualitatively	Significance
1	Own resources	Low
3	Additional resources from others	Medium
5	Most resources from others	High

Table 18. Factor of Available Capacity for i^{th} customer order

In case that available capacity can be immediately allocated from own resources based on the calculation in the department of planning, the capacity factor for i^{th} customer order takes the value 1; in case of limited resource and equipment and needed additional resources from others, the factor takes the value 3; and in the case when the calculated capacity for i^{th} customer order does not fulfil the requirements and most of the resources should be provided from others, the factor takes the value 5. These values are based on the insights and expertise of experienced professionals.

6. Discussion on Theoretical and Practical Advancements

6.1. Theoretical Basis

The proposed mathematical launching model builds upon established principles in production and operations management, extending current research by integrating various key factors into a cohesive framework for production scheduling and resource allocation.

6.1.1. Utilisation of Machines

Machine utilisation has traditionally been modeled with fixed capacities and schedules, which may not fully account for operational variability (Axsäter, 2022). Our model incorporates real-time data to dynamically adjust machine utilisation, reflecting operational fluctuations more accurately and improving scheduling precision. Recent studies highlight the effectiveness of real-time adjustments in optimizing machine performance (Barker, Yang & Chen, 2023).

6.1.2. Type of Machine

Existing models often generalize machine types without detailed differentiation, impacting production efficiency and costs (Krajewski, Ritzman & Malhotra, 2019). We classify machines in detail, enhancing capacity planning and resource allocation. This approach improves scheduling by considering the specific capabilities and constraints of each machine type, aligning with recent advances in machine-specific optimization.

6.1.3. Number of Employees Needed

Workforce planning in traditional models typically involves fixed employee requirements, which may not account for variations in job complexity and production demands (Heizer & Render, 2016). The model adjusts employee numbers based on job complexity and order size, resulting in optimized resource utilization and accurate scheduling.

6.1.4. Experience and Processing Time

Processing times are often treated as constants, ignoring the impact of employee experience (Choi, Kim & Park, 2024). By incorporating adjustments based on employee experience and historical performance data, our model provides more accurate processing time estimations, enhancing production scheduling. Recent studies underscore the role of experience in reducing processing times and improving operational efficiency.

6.1.5. Due Date and Release Date

Conventional models primarily focus on meeting due dates, with limited attention to release dates. Our model integrates both due dates and release dates into the scheduling process, aligning production schedules with customer expectations and operational constraints.

6.1.6. Customer Significance

Customer significance is often considered in priority-based scheduling, but detailed factors may be overlooked. Our model includes detailed customer significance factors such as payment history and potential for long-term cooperation, improving prioritization and resource allocation. Modern approaches emphasize the integration of comprehensive customer metrics for enhanced satisfaction.

6.1.7. Number of Parts and Purchase Time

Traditional models often treat the number of parts and purchase time separately (Cachon & Terwiesch, 2012). Our model integrates these factors, providing a comprehensive view of inventory requirements and purchasing timing. This approach improves inventory management and purchasing processes, as supported by recent research (Chen, Liu & Wang, 2024).

6.1.8. Material Handling

Material handling is typically modeled with fixed costs, overlooking variability. Our model accounts for the complexities of material handling and its impact on production schedules, enhancing material management and reducing delays. Recent studies highlight the importance of flexible material handling strategies (Lee, Lee & Wu, 2023).

6.2. Practical Advancements

The proposed model offers practical improvements over existing models.

6.2.1. Integration of Dynamic Factors

Many traditional models use static parameters that do not adapt to real-time conditions. By dynamically adjusting for factors such as machine utilisation, employee requirements, and material handling, our model enhances operational flexibility and responsiveness. Recent literature highlights the benefits of dynamic models in improving production efficiency (Kim, Lee & Chen, 2023).

6.2.2. Enhanced Accuracy

Traditional models often provide generalized estimates that may not capture the complexities of production processes. Incorporating detailed factors like machine type, employee experience, and customer significance leads to more accurate scheduling and resource planning. Modern approaches emphasize the need for detailed and accurate models to improve production outcomes.

6.2.3. Improved Resource Allocation

Resource allocation in many models is based on simplified assumptions. By integrating factors such as material availability and the complexity of securing additional resources, our model offers a more nuanced view of resource allocation, leading to more efficient production processes. Recent research underscores the importance of advanced resource allocation strategies (Xu, Zhang & Wang, 2024).

6.2.4. Real-Time Adjustments

Many models do not account for real-time adjustments, resulting in less adaptive production scheduling. Our model's ability to adjust for changes in processing time, purchase time, and material handling based on actual conditions allows for more responsive production management.

6.2.5. Customer-Centric Scheduling

Priority-based scheduling often lacks detailed customer-specific factors. By incorporating detailed customer significance factors, our model ensures high-priority orders are processed with the necessary attention, leading to improved customer satisfaction and loyalty.

In summary, our model represents a significant advancement in production scheduling and resource management by incorporating a comprehensive set of dynamic factors and offering practical improvements. This approach addresses the limitations of existing models and provides a more accurate and responsive framework for managing complex production environments.

7. Conclusion and Future Research

The findings of this study have significant implications for the integration of Industry 4.0 principles in Kosovo's enterprises. Industry 4.0 is characterized by the digitization and automation of manufacturing processes, utilizing technologies such as Internet of Things (IoT), artificial intelligence, big data, and cloud computing to create smart and interconnected production systems.

The low level of software application and the absence of planning and scheduling models in Kosovo's enterprises indicate a gap in adopting Industry 4.0 practices. The development of launching model, as proposed in this study, aligns with the objectives of Industry 4.0 by emphasizing the optimization of operations, efficient resource allocation, and improved production planning.

Industry 4.0 emphasizes the use of data to drive decision-making processes. The mathematical model incorporates data from various sources, including customer orders, product specifications, and resource availability, to optimize production operations. By leveraging data analytics and algorithms, the model enables enterprises to make more informed and data-driven decisions in real-time. Industry 4.0 promotes the automation and digitization of manufacturing processes. The proposed model can be integrated with digital systems and technologies to automate production planning and scheduling tasks. This integration enables seamless communication and coordination between different stages of the production process, minimizing manual intervention and improving overall efficiency. The mathematical model can be integrated with other digital tools and systems within an enterprise's manufacturing ecosystem, such as enterprise resource planning (ERP) systems, production monitoring systems, and supply chain management platforms. This integration enables seamless data exchange, real-time updates, and synchronized operations across different systems, leading to improved coordination and optimization of production processes. Finally, the mathematical model optimizes production planning and scheduling by considering various influential parameters, such as profitability, customer importance, and resource availability. By optimizing task prioritization, resource allocation, and production schedules, enterprises can achieve higher levels of efficiency, reduce waste, and enhance productivity.

By implementing software solutions and integrating digital technologies, Kosovo's enterprises can enhance their production processes, increase productivity, and achieve higher levels of efficiency. The use of an expert system, as suggested for future work, can further leverage Industry 4.0 principles by providing real-time decision support, enabling predictive maintenance, and optimizing job prioritization based on data-driven insights.

Our study highlights the critical need for manufacturing enterprises in Kosovo to address industry-specific challenges and adopt Industry 4.0 practices to remain competitive in today's globalized economy. The integration of Industry 4.0 principles into our model is paramount to its effectiveness in addressing the evolving needs of manufacturing enterprises. Specifically, our model incorporates data-driven decision-making processes and automation to optimize production sequences and resource allocation. By leveraging advanced mathematical algorithms and data analytics, the model enables manufacturing enterprises to make informed and data-driven decisions in real-time, thereby enhancing operational efficiency and competitiveness.

In conclusion, the findings of this study provide a stepping stone for Kosovo's enterprises to embrace Industry 4.0 practices. By incorporating digital technologies, implementing software solutions, and leveraging data-driven insights, these enterprises can enhance their competitiveness, adapt to changing market demands, and thrive in the era of Industry 4.0.

Acknowledgement

The authors would like to thank the owners of companies for sharing the data and their experiences, especially the owner of the Company "Bunsam" located in Kosovo.

Declaration of Conflicting Interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

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