

Inventory Policy Selection for Multi-Item and Multi-Supplier Systems Using Periodic and Continuous Models

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Abstract:

Purpose: The purpose of this paper is to select inventory policy model(s) that can minimize total inventory costs considering order lot size, reorder points, lead time, and maximum inventory levels. The evaluation of alternative policies incorporates both service level performance and implementation feasibility considerations. In this paper, the demand is probabilistic and follows normal distribution. Also, the lead time in the study has a potential to be accelerated through controllable replenishment adjustments and decisions. This capability enables the evaluation of trade-offs between replenishment responsiveness and inventory-related costs under probabilistic demand conditions. We consider possible shortages caused by unfulfilled customer demand due to stock shortages to be minimized. The scope of the problem involves multiple suppliers and multi-items.

Design/methodology/approach: Several normal-distributed probabilistic inventory policy models are proposed using perpetual/continuous inventory model (Q) and periodic inventory model (P). Furthermore, we have developed the periodic inventory model (P) for multi-item single-source (MISS) because of the potential for several different items to be supplied by the same supplier. This will be able to minimize the total inventory cost, considering that items are no longer treated individually. In addition, we also investigate the periodic and perpetual/continuous model with varying combination policy of shortage costs (*i.e.* backorder and lost sales policies).

Findings: The proposed inventory policy is able to save total costs consisting of ordering costs, holding costs, and shortage costs with a value of 68.40% for Supplier 1, 75.14% for Supplier 2, and 41.65% for Supplier 3. The research results also increased the service level with final service level 99.90% for Supplier 1, 99.91% for Supplier 2, and 97.29% for Supplier 3. The study also showed that the proposed model is sensitive to changes in demand, ordering costs, and holding costs parameters.

Originality/value: This paper offers a new approach, introducing P multi-item model with probabilistic demand in multiple supplier systems considering backorder, lost sales and controllable lead-time acceleration capability. Furthermore, the developed model is compared with existing inventory models to see its effectiveness. The existing models are P -model, Q -model, multi-item model, and lead time reduction model. Of the three suppliers, the integrated P model and multi-item model is the best policy for two suppliers in terms of total inventory cost and service level aspects.

Keywords: inventory policy, periodic inventory, perpetual/continuous inventory, multi-item inventory, multiple suppliers, probabilistic demand, backorder and lost sales

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1. Introduction

Today's business environment continues to evolve to become more complex and competitive, which makes companies constantly look for strategies to generate profits and remain relevant among their competitors. As a result, companies are challenged to design an effective and well-integrated supply chain system. The main concept of the supply chain (SC) is to build good relations and coordination between suppliers, production, warehouses, retailers, and customers to provide more accurate customer satisfaction. Inventory level chain management aims to synchronize between capacities supplier with consumer demand characterized by balance unit cost, inventory level, rejected items, the low and high customer service (Stevens & Johnson, 2016).

Inventory management issues are critical challenges in the inventory system to help optimize the inventory level and reduce cost while still meeting customer demands (Al-Shukaili et al., 2023). In the context of inventory management, there are two main considerations: (1) determining how much inventory should be maintained in stock to ensure sufficient supplies and to minimize unnecessary inventory investments, and (2) determining how the frequency of inventory should be replenished. Demand can be highly unpredictable and crucial in firms' ordering and stocking decisions (Eroglu et al., 2023; Jehan, 2020). Customer demand uncertainty creates challenges to the planning process, and if not addressed properly can lead to shortages or surpluses of raw materials (Dadaneh et al., 2023).

One of the common inventory management issues is lost sales due to the inability to meet customer demand on time. It might be more profitable to lose demand rather than fulfill it. There are four aspects that contribute to total sales loss, i.e. (1) the workers cannot operate the system properly; (2) fluctuations in demand, product returns to vendors, and expired products; (3) error in the warehousing system and transportation equipment; and (4) the order quantity, time, and the number of shipments are incorrect, and there is a lack of stock. The uncertainty in demand becomes even more complex when organizations must manage multiple items simultaneously, each with different demand characteristics and priority levels. In addition, managing multiple suppliers adds an additional layer of complexity. Reliance on multiple suppliers to meet inventory needs can increase the risk of supply disruptions. Different suppliers' delivery times can lead to mismatches between available inventory and actual demand. Therefore, organizations need to design inventory policies that are not only responsive to demand but also adaptive to changing supply conditions. The explanation above indicates improper company policy.

This paper addresses a challenge often encountered by inventory managers that is to minimize the costs of ordering, holding and potential shortages while meeting the customer demand, while incorporating service performance and implementation feasibility considerations. In a majority of sectors, it is required to consider demand uncertainties, which may be the cause of shortages (out of stock) or having too much inventory on hand (over stock). In this study we focus on the practical situations in which an inventory manager is tasked to handle multiple items that are procured from multiple suppliers and each item follows a unique demand pattern and lead time. This is especially important in an environment where demand is probabilistic and lead times can be accelerated. Therefore, this study was conducted to determine the optimal inventory policy. The policy includes product order lot size, reorder points, lead time, and maximum inventory levels.

Despite the extensive literature on probabilistic Q and P inventory models, most studies have either focused on single-item systems or on the independent effects of multi-item decisions with no explicit approach for coordinating replenishment at the supplier level. Furthermore, although backorder and lost sales policies have been studied individually, limited research combines the shortage policies of these strategies within a unified multi-item and multi-supplier probabilistic framework. In addition, practical replenishment systems may allow controllable

lead-time acceleration through operational prioritization, expedited transportation, or such activities. However, most existing probabilistic inventory studies assume fixed lead times and do not explicitly incorporate accelerated lead-time capability within coordinated multi-item replenishment structures.

This study addresses these limitations by developing a supplier-level coordinated periodic review model for multi-item systems under probabilistic demand. The proposed framework integrates (i) multi-item single-source coordination, (ii) probabilistic demand with service-level control, (iii) alternative shortage policies (backorder and lost sales), (iv) controllable lead-time acceleration capability, and (v) supplier-level coordinated replenishment into a unified decision architecture. The objective is not only to minimize total inventory cost but also to provide a structured mechanism for policy differentiation across suppliers.

The theoretical contribution of this study lies not in introducing a completely new inventory model, but in reformulating the selection of inventory policy from independent item-level optimization to supplier-level coordinated decision-making under probabilistic demand. Unlike traditional inventory approaches that evaluate replenishment decisions separately for each item, the proposed framework integrates multi-item coordination at the supplier level while simultaneously comparing periodic review policies, continuous review policies, shortage treatments (backorder and lost sales), and implementation-oriented managerial considerations within a unified decision structure. Therefore, the study contributes to the coordinated replenishment literature and to the practical selection of inventory policies under supplier heterogeneity.

In particular, the theoretical novelty of the proposed approach is threefold. First, the study transforms the optimization structure of periodic review inventory systems from item-level independence to a supplier-level coordinated decision making, where the replenishment interval is shared by all items from the same supplier and optimized jointly. This structural change captures the trade-offs between items in ordering, holding and shortage costs which are not possible in conventional item-level models. Second, the proposed model is different from the existing multi-item inventory models in the sense that it considers both backorder and lost-sales shortage policies at the same time in a probabilistic multi-item and multi-supplier framework. Most of the existing models for multi-item systems assume independent shortage policies or deterministic demand, thus limiting their applicability to real supply chain environments where the presence of demand uncertainty and the heterogeneity of suppliers are commonplace. Third, unlike previous extensions

The remaining content is organized as follows. Section Literature Review reviews some of the most relevant studies. Section System Description is the description of the studied system. Section 4 lists the notation. Section 5 builds inventory policy model while section 6 shows a numerical example of the developed model. The results and its analysis are presented in Section 7. Section 8 is catered at presenting graphical to show the sensitivity of the developed model regarding some parameters. Finally, the conclusion along with some scope for future research are presented in Section 9.

2. Literature Review

Inventory management has been a persistent focal point within industrial engineering and operational research. Due to its pivotal function in business operations, the inventory planning process determines the most advantageous timing and quantity for orders (Moslemi & Zandieh, 2011). Retailers are aware of the cost of losing sales, along with the costs of inactive inventory (Ramos et al., 2020). While retailers deal with rapidly moving items, they always need to adjust to new supply chain strategies that will increase the speed of products through the supply chain and increase the accuracy of inventory management (Chow et al., 2008; Sohel et al., 2016; Conceição et al., 2021).

Inventory systems concerning inventory review policy (perpetual or periodic), the reorder level (zero or non-zero), the demand (deterministic or stochastic; constant or non-constant), the time horizon (finite or infinite) have been studied and modified in many literatures nowadays (Karakatsoulis & Skouri, 2023). Regarding the periodic inventory review policy, Gupta and Starr (2014) explained how the quantity ordered is based on the current inventory level when the record is read. As a result, in a periodic system, while the interval between orders remains fixed, the order amount varies. Other studies related to several model inventories and their characteristics is seen in Table 1.

Author(s)	Reorder	Demand		Lead time	Replenishment policy		Product		SL	Decision variables
		Det	Prob		Back-order	Lost sales	Single	Multi		
(Wu & Lin, 2004)	✓		✓	Constant	✓		✓		–	The ordering cost, safety factor k
P-model (Gupta & Starr, 2014)	✓		✓	Constant	✓	✓	✓		✓	The period between orders and the maximum expected inventory
Q-model (Gupta & Starr, 2014)	✓		✓	Constant	✓	✓	✓		✓	Lot size for each order and reorder point
(Heydari, 2014)	✓	✓		Stochastic		✓		✓	✓	Customer Service level, Lead time variance
(Priyan & Uthayakumar, 2015)	✓		✓	Variable	✓	✓	✓		✓	Lead time, lost sales rate, and order processing cost
(Fergany, 2016)	✓		✓	Constant	✓	✓		✓	–	The time between orders and the number of items ordered for each item
(Roni et al., 2016)	✓		✓	Constant	✓		✓		✓	Regular reorder level, emergency reorder level, regular order quantity, and emergency order quantity
(Heydari et al., 2016)	✓	✓		Stochastic	✓		✓		✓	Reorder points coefficients, Ordering multipliers of the distributor and the producer
(Escalona et al., 2018)	✓		✓	Constant	✓		✓	✓	✓	Allocation
(Alawneh & Zhang, 2018)	✓		✓	Uncertain	✓			✓	✓	Order quantity and reorder point
(Abbaspour et al., 2022)	✓		✓	Variable		✓	✓		–	Reorder point, Reorder quantity, Number of servers, Price, time interval of order, distance traveled
(Haghi-Motlagh & Reza-Nasiri, 2023)	–		✓	–	✓	✓	✓		–	Price offered by supplier, extra profits, final price of selling the product, consumer price, and order quantity
(Gökbayrak & Kayış, 2023)	–		✓	–	✓	✓	✓		✓	Inventory level

Author(s)	Reorder	Demand		Lead time	Replenishment policy		Product		SL	Decision variables
		Det	Prob		Back-order	Lost sales	Single	Multi		
(Wood & Çetinkaya, 2024)	✓		✓	Constant	✓	✓	✓		✓	Stockout policy selection
(Azimian & Aouni, 2025)	✓		✓	Constant	✓			✓	✓	Multi-item order quantity
(Barrera-Sánchez & García-Cáceres, 2025)	✓		✓	Constant	✓			✓	✓	Inventory level, replenishment policy
This study	✓	✓	✓	Can be accelerated	✓	✓	✓	✓	✓	Time between orders and the maximum inventory level

Table 1. Review of inventory policy studies in literature

Although the existing inventory models offer foundational insights, they often shortsighted in addressing the complexities of multi-supplier and multi-item systems. This paper points out the limitations of traditional models, perpetual inventory model (Q) and periodic inventory model (P) for examples, which have been developed for single product, making them less effective in multi-item inventory systems without significant adjustment. Additionally, the interrelation of order costs, holdings costs, shortage costs, crashing costs, and service levels with probabilistic demand presents a further theoretical challenges that have not been extensively analyzed in the literature.

Recent studies have increasingly emphasized the importance of inventory coordination under probabilistic demand and shortage uncertainty. Wood and Çetinkaya (2024) compared several approaches for handling stockout scenarios under stochastic inventory systems and highlighted that different shortage treatments, such as backorders and lost sales, may significantly affect inventory cost and service-level performance. In the context of multi-item inventory optimization, Azimian and Aouni (2025) proposed a stochastic goal programming approach for determining optimal order quantities under uncertain demand and demonstrated the importance of incorporating decision-maker considerations into inventory policy selection. Furthermore, Barrera-Sánchez and García-Cáceres (2025) developed a stochastic inventory planning framework for multi-product retail systems with deterministic lead time and supplier-related constraints, emphasizing the need for coordinated inventory decision-making across multiple products. Despite these recent developments, limited studies simultaneously integrate probabilistic demand, alternative shortage policies (backorder and lost sales), supplier-level multi-item coordination, and implementation-oriented policy selection within a unified periodic and continuous review inventory framework. Therefore, this study proposes an integrated inventory policy selection framework for multi-item and multi-supplier systems considering both quantitative inventory performance and managerial implementation feasibility.

Although previous studies have addressed stochastic inventory systems, most existing models either focus on item-level optimization, single-policy structures, or isolated shortage treatments. Limited attention has been given to supplier-level coordinated replenishment under probabilistic demand while simultaneously evaluating alternative shortage policies and implementation feasibility considerations. Furthermore, many traditional models assume homogeneous replenishment behavior across items, which may not adequately represent operational conditions involving heterogeneous suppliers and varying demand characteristics.

This paper presents a new approach that incorporates perpetual inventory model (Q) and periodic inventory model (P) models with probabilistic demand and considers both backorder and lost sales, which this paper aims to fill these gaps. The objective of the proposed model is to minimize the total inventory costs as well as to improve the service levels, thereby providing a decision support tool to inventory managers for tackling the complexity of modern supply chains.

A critical analysis of these studies reveals three important patterns, which are relevant to the present study. First, in the context of coordinated replenishment systems, lead-time coordination and reorder point adjustment have been studied in the literature (e.g., Heydari, 2014; Heydari et al., 2016) but are restricted to single-item or two-echelon settings and do not consider joint ordering decisions over multiple items from the same supplier. Similarly, Roni et al. (2016) and Escalona et al. (2018) also consider emergency replenishment and allocation mechanisms, but their frameworks do not generalize to supplier-level coordination of multi-item replenishment intervals. Second, in terms of multi-item inventory optimization, the main multi-item contributions in the reviewed literature are from Fergany (2016) and Chary (2012), but both assume deterministic demand and treat items independently, which limits their applicability to environments characterized by demand uncertainty and supplier heterogeneity. Third, most of the stochastic inventory models reviewed are developed for the single-item single-supplier settings. In the context of probabilistic inventory models under supplier heterogeneity such as Wu and Lin (2004), Priyan and Uthayakumar (2015) and Alawneh and Zhang (2018). Few studies consider multiple products (e.g. Azimian & Aouni, 2025; Barrera-Sánchez & García-Cáceres, 2025) and evaluate simultaneously alternative shortage policies in a coordinated multiple-supplier replenishment framework. The above observations collectively imply that the combined area of supplier-level coordination, probabilistic demand, multi-item structure and heterogeneous treatment of shortages is under-researched in the existing literature.

In this paper, four inventory models are employed, 1) Perpetual inventory model (fixed quantity) (Q model), (2) Multi-item inventory model, (3) Periodic inventory model (fixed time) (P model), and (4) Lead time reduction inventory model by Priyan and Uthayakumar (2015). However, these models provide basic understanding of inventory management, but they may not be capable to deal with the complicity to be faced in present supply chain situations. For example, Priyan and Uthayakumar (2015) stress the importance of our capacity to consolidate variability in lead time variability and the potential for shortages into inventory policies. The findings from this study demonstrate how the static assumptions by Q and P model result in inefficiencies when organizations are faced with varying patterns of demand, multiple products, diverse supplier relationship.

In light of these considerations, the need for adjustments and further development of the Q and P models becomes increasingly pertinent. The presence of more advanced features: probabilistic demand, multiple item management and the ability to have several suppliers, is essential to develop an efficient inventory management system. The identification of such gaps is important as the designs of improved inventory models would then equip firms with the repertoire of models necessary to cope with the complexity of supply chain activities.

Priyan and Uthayakumar's (2015) reference model is a development of the Q inventory model with lead time reduction. This model also accommodates the possibility of a shortage of items in accordance with the actual conditions of companies that also experience this. Lack of items can occur due to damage to items, non-conformity of items, or other errors in the process of ordering and shipping items. This model also has the characteristics of repeat orders, which makes it more responsive to large demands. Using this model requires a more intensive administration level because inventory conditions need to be considered periodically so that orders can be made when they have reached a certain inventory level. In this paper, we do this by comparing inventory models from the literature studies that have been collected to determine optimal inventory policies that can minimize total inventory costs. There are four models chosen because of their suitability to the research problem (see Table 2).

The Priyan and Uthayakumar (2015) model is used as a reference model because the design time in the study system has the potential to be accelerated by adding *crashing* costs. In addition, the model adjustment that must be made is an adjustment to the P inventory model for multi-items with possible shortages. The characteristics of multi-item models need to be considered given the potential for items to be supplied by the same supplier. After the reference model has been adjusted, it will proceed with the data processing process. Thus, there are models that will be compared with performance results, namely the P inventory model for multi-items and the Q inventory model Priyan and Uthayakumar (2015). In addition to the adjusted model, data processing is also carried out with the Q model with the possibility of loss as a comparison.

From these four inventory models, we need to develop the periodic review (fixed time) inventory model by changing the number of ordered items from unit/single item to multi-items because of the potential for items to

be supplied by the same supplier. In doing so, we integrated the P model with the multi-item single source. The existence of multi-items characteristic does not change the inventory cost component in the object of study. However, the total inventory cost for each item will be grouped according to its supplier. We develop this model to better represent the real conditions of the case discussed in this paper.

Characteristics	Study system	Model Q (Gupta & Starr, 2014)	Model P (Gupta & Starr, 2014)	Multi-Item (Chary, 2012)	Accelerated Lead Time (Priyan and Uthayakumar, 2015)
Orders execution	If quantity reaches the reorder point	If quantity reaches the reorder point	At fixed intervals of time	At a certain order time interval	If quantity reaches the reorder point
Order lot size	Free amounts	Always fixed	The maximum difference between the desired inventory and the amount of inventory available	The maximum difference between the desired inventory and the amount of inventory available	Uncertain
Demand	Probabilistic	Probabilistic	Probabilistic	Deterministic	Probabilistic
Lead time	Can be accelerated	Constant	Constant	Constant	Variable
Replenishment policy	Lost sales/Backorder	Lost sales/Backorder	Lost sales/Backorder	Lost sales/Backorder	Lost sales/Backorder
Number of items ordered	Single/Multi	Single	Single	Multi	Single
Purpose function	Minimization of total inventory costs	Minimization of total inventory costs	Minimization of total inventory costs	Minimization of total inventory costs	Minimization of total inventory costs

Table 2. Comparison of the study system with the reference model

The distinctive capability of the proposed model to adapt to various scenarios within the same organization is emphasized. This adaptability is what drives the learning in the model. Featuring at least three inventory models adapted to the specific circumstances of each supplier, the proposed model introduces a level of flexibility that traditional inventory management strategies often lack. This characteristic enables a more subtle and efficient inventory management system that can adapt to the varied needs of different suppliers.

The integration of three inventory models is crucial because it enables an integrated inventory management strategy with consideration on the variation of demand, lead time and cost due to the difference of the supplier. For instance, the Q model which minimizes holding costs of an order quantity may be applicable with a supplier with a stable demand. For the suppliers with non-constant demands the P model is more useful instead, since it allows for periodic reviews and adjustments of orders with stock on hand at any given time. In addition, multi-item model can be used to control the multi-product by the suppliers with high variation of lead-time and demand, maintaining service levels while controlling costs.

The proposed framework differs from traditional Q and P models by internalizing cross-item ordering interactions at the supplier level. Traditional models usually work under static assumptions which fail to consider the dynamic interrelationship between items and suppliers. Furthermore, though Priyan and Uthayakumar's (2015) model incorporates a reduction of lead time, it still cannot persuade different suppliers such as effectively as the integrated approach. This proposal enables the combination of various models tailored to supplier conditions which not only minimizes total inventory costs but also increases the service levels.

The present study was motivated by three particular gaps in the extant literature. First, although multi-item inventory models have been developed (Fergany, 2016; Chary, 2012), replenishment decisions in these models are generally made independently for each item, even if the items are from the same supplier. This item-level

independence neglects possible coordination effects at the supplier level which can significantly reduce ordering costs and improve replenishment synchronization. The model proposed in this paper attempts to fill this gap by optimising a common replenishment interval at the supplier level thereby establishing a structural interdependence among all items sourced from the same source. Second, the treatment of shortages, such as backorders and lost sales, under single-item probabilistic models has been studied extensively (Wu & Lin, 2004; Priyan & Uthayakumar, 2015), but there is very little research on the comparative evaluation of such treatments in a multi-item and multi-supplier setting. The interaction of shortage policy selection with supplier heterogeneity and multi-item coordination is poorly understood. The proposed model bridges this gap by treating both backorder and lost-sales policies as equivalent options within one integrated decision framework. Third, while controllable lead-time reduction has been considered in Q -model extensions (Wu & Lin, 2004; Priyan & Uthayakumar, 2015), these models are single-item oriented and do not study the impact of lead-time acceleration on coordinated multi-item replenishment decisions. The proposed model takes the advantage by embedding the controllable lead-time acceleration capability in the supplier-level coordinated periodic review structure. It allows for an explicit cost-responsiveness trade-off evaluation under probabilistic demand.

To the best of authors' knowledge, there is no existing study that integrates: (i) supplier-level multi-item coordination; (ii) probabilistic demand; (iii) alternative shortage policies (backorder and lost sales); and (iv) controllable lead-time acceleration in a unified periodic and continuous review inventory decision framework applicable to multi-supplier systems. This paper aims to address this gap by proposing a structured supplier-level decision architecture that minimizes total inventory cost and supports the selection of implementation-oriented policies under heterogeneous supplier conditions.

3. System Description

This study only examines problems related to the inventory system of a company. Looking at the form and type of inventory, the inventory studied in this case is inventory in the form of finished goods (Figure 1). Inventory is needed to ensure the fulfillment of needs and as a motive to guard against uncertainty in demand. Therefore, operating stocks and safety stocks are needed to overcome such uncertainties.

The problem of inventory policy at the company is recurring decision making with continuous ordering. Product demand comes from end consumers who purchase products through retailers. The retailer's request will be forwarded and processed by the company's depot warehouse. The problem that occurs is that the depot warehouse cannot determine the size of the order lot.

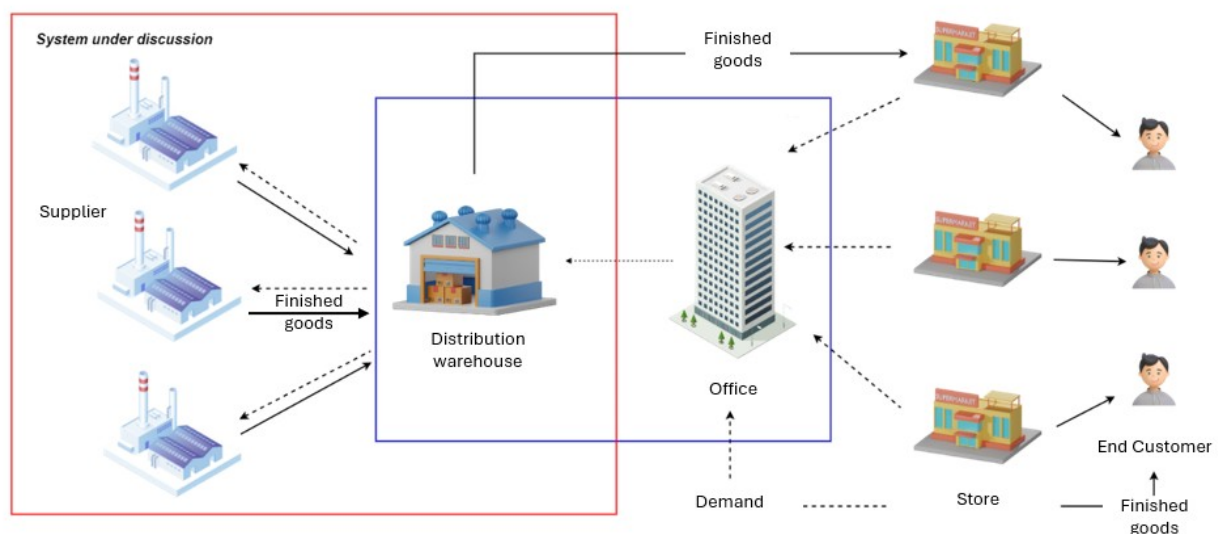


Figure 1. Structural aspects of the company's inventory system

4. Notations

The notations presented below are used throughout the study.

Notation	Description	Unit
D	Average demand per period	Unit/time
p	Purchase price per unit	USD /unit
A	Ordering cost per order	USD
b	Holding cost per unit per period	USD /unit/time
L	The lead-time between the placement of an order and its receipt	Time
π	Fix penalty cost per unit short	USD /unit
s	The standard deviation of demand per period	Unit/time
α	Bias factor which is the expected amount received	–
$\tilde{z}_\alpha$	The value of z at normal distribution with probability α , represent critical values of standard normal distribution	–
$f(\tilde{z}_\alpha)$	Normal distribution ordinate values at the level $\tilde{z}_\alpha$	–
$\Psi(\tilde{z}_\alpha)$	The partial expectation of normal distribution at the level of $\tilde{z}_\alpha$. The expected value of the normal distribution when it is considered only specific portion of distribution, typically either tan the right or left side. Often used in teh contexto f inventory management. Other words it is called probability density function.	–
D_L	Demand during lead time	Unit
S_L	The standard deviation of demand over the lead time	Unit
ss	Safety stock	Unit
N	Expected shortage quantity during the protection interval (lead time + time between order interval)	Unit
m	Expected amount of inventory stored during the planning horizon	Unit
O_b	The expected purchasing cost during the planning horizon	USD
O_p	The expected ordering cost during the planning horizon	USD
O_s	The expected holding cost during the planning horizon	USD
O_k	The expected shortage cost during the planning horizon	USD
Q_o	Order lot size per order	Unit
C_o	The ordering cost per order	USD
Ch	Holding cost per order	USD/unit/time
Cs	Shortage cost per unit	USD/unit
r	The reorder point of the i th item per period	Unit
O_T	The total cost of inventory over the planning horizon	USD
R	Maximum inventory level	Unit
i	Item index ($i = 1, 2, 3, \dots, n$)	1
D_i	Demand for items during the planning horizon i	Unit
p_i	Purchase price of items per unit for items i	USD /unit
b_i	Holding cost per unit per period for items i	USD /unit
T	Time interval between orders	Time

Notation	Description	Unit
Q_i	The order quantity of the i th item per period	Unit
A_0	The cost of ordering each time an order (before any investment)	USD
Y	Received quantity	Unit
α	The probability of a shortage occurring	–
β	A constant real number selected to provide the best fit of estimated expected cost function	–
π_0	Marginal profit per unit	USD /unit
X	Request during lead time with average DL and standard deviation $\sigma\sqrt{L}$	Unit
$E(\cdot)$	Expected value	

Table 3. Notations

5. Model Development

This section provides the development and structural distinction of the inventory models in this study. It initially describes the contrast between the periodic and perpetual review systems, followed by the mathematical formulation of the periodic (P) model, the perpetual (Q) model with fixed order quantity, and the lead time reduction model adapted from Priyan and Uthayakumar (2015). The section ends with the fundamentals and the limitations of the proposed models.

5.1. Model Differentiation

While periodic review (P) and multi-item single-source (MISS) models have been extensively studied, the proposed formulation introduces structural modifications in two aspects. In classical MISS models, replenishment parameters are typically determined independently for each item, even when items originate from the same supplier. The ordering cost may be aggregated, but decision variables remain item-specific. In contrast, the proposed model optimizes a common ordering interval at the supplier level, creating interdependence among items supplied by the same source. Second, the cost aggregation logic is reformulated. Instead of minimizing the total cost per item and summing the results, the model minimizes the expected total inventory cost aggregated at the supplier. This structural shift internalizes cross-item trade-offs in holding, ordering, and shortage costs. Therefore, the theoretical contribution of this study lies not in redefining probabilistic demand assumptions, but in reformulating the optimization structure from item-level independence to supplier-level coordinated decision-making under uncertainty.

5.2. Periodic Inventory Model

The development of the P model for multi-item single-source is carried out considering that several items come from the same supplier, so the total inventory cost can be minimized by ordering simultaneously. Determining the optimal order interval for each supplier is essential to placing orders simultaneously. Therefore, the multi-item model has characteristics similar to the P model as follows.

1. Constant interval of order time intervals.
2. The size of the order lot is the maximum difference between the desired inventory and the amount of inventory available at the time the order is placed.

Several assumptions are used in the P multi-item inventory model, as follows.

1. Demand during the planning horizon is probabilistic and normally distributed with average demand and standard deviation.
2. The time between orders is constant each time an order is placed.
3. The order lot size is the difference between the desired maximum inventory and the existing inventory at the time the order is placed.
4. Products ordered will come simultaneously with the time of preparation.

5. The price of items is constant with respect to the quantity of items ordered and the time the order was made.
6. The cost of ordering each time is constant.
7. Storage costs are proportional to the items' price and the time they are stored.
8. The cost of inventory shortages is proportional to the number of items that cannot be serviced or proportional to the service time.

There are three components of P multi-item as follows.

1. Performance criteria: The performance criteria of the developed model are to find the minimum value of the expected total inventory cost for each supplier. The expected total inventory cost is calculated for each supplier. It consists of the purchase cost, order cost, holding cost, and shortage cost of each item ordered through the same supplier.
2. Decision variables: The decision variables for the developed model are the time between orders and the maximum inventory level for all items in each supplier.
3. Parameters: The parameters used in the developed model are the price of item per unit, the ordering cost for each order, the storage cost for items per unit of time, and the shortage cost for each item ordered through the same supplier.

There are several multi-item model equations to be formulated, as follows.

5.2.1. Purchasing Cost

Purchasing cost is the cost incurred to purchase items during the planning horizon. The multi-item model has normal distribution. This is reflected by probability density function ($\Psi(z_d)$) and average demand (D). The variation is reflected by standard deviation (S_L) of demand. The shortage cost has been considered in O_{kj} (The expected shortage cost per period). Purchasing cost is obtained by adding up all the products and the number of items purchased during the planning horizon with the purchase price per unit of items. The purchasing cost equation can be seen in Equation (1).

$$E(O_{bj}) = \sum_{i \in j} D_i p_i \quad (1)$$

5.2.2. Ordering Cost

Ordering cost represents the expected ordering expenditure normalized over the planning horizon through time interval between orders T_j , such that all inventory cost components are evaluated on a comparable expected-time basis. This ordering cost equation has a similar formulation to the P model by dividing the amount of ordering cost by the time between orders. The ordering cost equation for each supplier is shown in Equation (2).

$$E(O_{pj}) = \frac{Co_j}{T_j} \quad (2)$$

Where T_j represents the time interval between orders expressed in planning-horizon time units.

5.2.3. Holding Cost

The holding cost in the developed model is obtained by multiplying the expected amount of inventory by the holding cost per unit during the planning horizon. The expected amount of inventory held during the planning horizon depends on how to deal with inventory shortages, namely backorders and lost sales.

a) Holding cost of the developed model with backorder

In the case of backorders, customers are willing to wait until the items are available. In the backorder case, unmet demand is temporarily recorded as backordered demand within the inventory position representation until replenishment arrives. This representation does not imply physically negative on-hand inventory. Therefore, holding

cost is associated only with expected on-hand inventory, while expected backordered quantities are incorporated separately within the shortage cost component. The expected amount of inventory during the planning horizon can be seen in Equation (3).

$$m_i = \frac{D_i T_j}{2} + ss_i \quad (3)$$

In the periodic review system, the maximum inventory level is intended to cover expected demand during the protection interval, which consists of the time interval between orders T_j and replenishment lead time L_i . Therefore, safety stock is defined as the inventory buffer above the expected demand during the protection interval, as shown in Equation (4).

$$ss_i = R_i - D_i(T_j + L_i) \quad (4)$$

The expected holding cost in the developed model with backorder can be seen through Equation (5).

$$E(O_{sj}) = \sum_{i \in j} ch_i \left(\frac{D_i T_j}{2} + ss_i \right) \quad (5)$$

b) Holding cost of the developed model with lost sales

In this case, customers are not willing to wait, so the shortage that occurs causes a loss of sales for the item's provider. This situation does not allow for negative inventory. The expected positive inventory during the planning horizon can be explained through equation (6)-(10).

The expected shortage quantity under probabilistic demand is defined as:

$$N_i = E[(X_i - R_i)^+] \quad (6)$$

$$E[(X_i - R_i)^+] = \int_{R_i}^{\infty} (x - R_i) f(x) dx \quad (7)$$

Where X_i represents demand during the protection interval $T_j + L_i$. Assuming normally distributed demand, the expected shortage quantity can be expressed analytically as shown in Equation (13).

Since lost-sales systems do not allow negative physical inventory, holding cost is evaluated based on expected positive on-hand inventory:

$$E[(R_i - X_i)^+] = R_i - E[X_i] + E[(X_i - R_i)^+] \quad (8)$$

And:

$$E[X_i] = D_i(T_j + L_i) \quad (9)$$

The expected positive inventory becomes:

$$E[(R_i - X_i)^+] = R_i - D_i(T_j + L_i) + N_i \quad (10)$$

Therefore, the term $+N_i$ represents a mathematical correction term arising from the expected positive inventory formulation rather than a holding cost charged directly to shortage quantities. Unlike Equation (4), which defines safety stock conventionally as inventory buffer above expected protection-interval demand, Equations (6)-(10) represent the expected positive on-hand inventory formulation under lost-sales condition. The expected holding cost in the developed model with lost sales can be seen through Equation (11).

$$E(O_{sj}) = \sum_{i \in j} Ch_i \left(R_i - D_{Li} - \frac{D_i T_j}{2} + N_i \right) \quad (11)$$

5.2.4. Shortage Cost

The value of shortage cost is obtained by multiplying the expected amount of shortage by the shortage cost per unit. The amount of shortage cost will be added up for all items supplied by the same supplier. The expected shortage cost equation is shown in Equation (12).

$$E(O_{kj}) = \sum_{i \in j} \frac{Cs_i N_i}{T_j} \quad (12)$$

The magnitude of the expected shortage is written in Equation (13).

$$N_i = s_i \sqrt{T_j + L_i} [f(z_{ai}) - z_{ai} \Psi(z_{ai})] \quad (13)$$

5.2.5. Total Inventory Cost of the Developed Model

Total inventory cost is obtained by adding up all inventory cost components for each item originating from the same supplier. The expected total inventory cost equation is shown in Equation (14) and Equation (11).

$$E(O_{Tj}) = E(O_{bj}) + E(O_{pj}) + E(O_{sj}) + E(O_{kj}) \quad (14)$$

$$E(O_{Tj}) = \sum_{i \in j} D_i p_i + \frac{Co_i}{T_j} + \sum_{i \in j} Ch_i \left(\frac{D_i T_j}{2} + ss_i \right) + \sum_{i \in j} \frac{Cs_i N_i}{T_j} \quad (15)$$

5.2.6. Decision Variable

The expected amount of inventory stored during the planning horizon depends on addressing inventory shortages, such as backorder and lost sales. Thus, each inventory model is divided into these two categories for the rest of the paper.

a) Backorder

The magnitude of the decision variable of the adjustment model with backorder is obtained by the process of differentiating the total cost in Equation (15) against the time between orders (T) and the maximum inventory point (R). The minimum total cost requirement can be seen in Equation (16) and Equation (17).

$$\frac{dE(O_{Tj})}{dT} = 0 \xrightarrow{\text{result}} T_j^* = \text{cannot be obtained} \quad (16)$$

$$\frac{dE(O_{Tj})}{dR} = 0 \xrightarrow{\text{result}} a_i = \frac{Ch_i T_j}{Cs_i} \quad (17)$$

The maximum inventory point for each item i is shown in Equation (18).

$$R_i = D_i (L_i + T_j) + z_{ai} s_i \sqrt{T_j + L_i} \quad (18)$$

b) Lost sales

The magnitude of the decision variable of the adjustment model with lost sales is obtained by the process of differentiating the total cost in Equation (15) against the time between orders (T) and the maximum inventory point (R). The minimum total cost requirement can be seen in Equation (19) and Equation (20).

$$\frac{dE(O_{Tj})}{dT} = 0 \xrightarrow{\text{result}} T_j^* = \text{cannot be obtained} \quad (19)$$

$$\frac{dE(O_{Tj})}{dR} = 0 \xrightarrow{\text{result}} a_i = \frac{Ch_i T_j}{Cs_i + Ch_i T_j} \quad (20)$$

The search for the value of decision variables in this model adjustment is carried out iteratively to achieve optimal results. The steps to find a solution are described below.

1. Calculate the value T_j with $T_j = \sqrt{\frac{2A}{\sum_{i=1}^n h_i D_i}}$.
2. Calculate the value a_i for each item- i , which comes from the supplier- j .
3. Calculate R_j for each item- j , which comes from the supplier- j .
4. Calculate N_i for each item- i , which comes from the supplier- j .
5. Calculate the total cost from suppliers- j in the first iteration.
6. Iterate through the calculation by adding and subtracting ΔT . Repeat Step 2 through Step 5 for each item- i which comes from the supplier- j .
7. Compare the total cost value of the first iteration, the increment iteration, and the subtraction iteration. Iteration stops when the total cost value of the initial inventory is the smallest. If the smallest total inventory cost comes from the increment iteration T_i , then the iteration continues by adding values T and repeating Step 2 through Step 5. Iteration stops when the increment iteration no longer results in a smaller total inventory cost and the optimum solution is the smallest current total inventory cost. The same is done for the iteration of value reduction T .

5.3. Perpetual Inventory Model

The perpetual fixed quantity or probabilistic Q inventory model has two basic characteristics (Gupta & Starr, 2014). The following are the basic characteristics of the Q inventory model.

1. Order lot sizes are always fixed for each order.
2. An order is made if the inventory quantity reaches the reorder point.

Several assumptions are used in the Q model. According to Gupta and Starr (2014) and Chary (2012), the assumptions used in the Q model are as follows.

1. Demand during the planning horizon is probabilistic and normally distributed with average demand (D) and standard deviation (S).
2. Order lot size (Q) is constant, items will arrive simultaneously with lead time (L), and orders are made if the amount of inventory has reached the reorder point (r).
3. The price of items (p) is constant with respect to the number of items and time of order.
4. The cost of ordering (A) is constant for each order, and the cost of holding (h) is proportional to the items' price and storage length.
5. The cost of inventory shortages is proportional to the number of requests for items that cannot be fulfilled or proportional to the service time.

Total inventory cost is obtained by adding up all inventory cost components for each item originating from the same supplier. The total inventory cost equation is shown in Equation (21) and Equation (22) (Bahagia, 2006).

$$O_T = O_b + O_p + O_s + O_k \quad (21)$$

$$O_T = Dp + \frac{DA}{Q_0} + h\left(\frac{Q_0}{2} + ss\right) + \frac{D}{Q_0} x \pi x N \quad (22)$$

With the holding purchase cost (O_b), order cost (O_p), holding cost (O_s), and shortage cost (O_k) as follows.

$$O_b = D x p \quad (23)$$

$$O_p = \frac{D}{Q_0} \times A \quad (24)$$

$$O_s (\text{lost sales}) = h \left(\frac{Q_0}{2} + r - D_L \right) \quad (25)$$

$$O_s (\text{backorder}) = h \left(\frac{Q_0}{2} + r - D_L + N \right) \quad (26)$$

$$O_k = \frac{D}{Q_0} \times \pi \times N \quad (27)$$

The following explains the steps for determining the Q model solution.

1. Set the values $i = 1$ and $r = 0$, with i indicating the first iteration and the initial reorder point values for comparison.
2. If $i = 1$, calculate $Q_1 = \sqrt{\frac{2AD}{h}}$, if $i \neq 1$ then proceed to Step 3.
3. Calculate Q_i .
4. Calculate α .
5. Calculate r .
6. Set $i = i + 1$.
7. Calculate N_i .
8. If then, proceed to Step 9; if not, then repeat Step 8.
9. Set $Q_1 = Q_{i-1}$ and $r_1 = r_{i-1}$.
10. Calculate O_T .

5.4. Lead Time Reduction Model

The inventory model will accommodate lead time so that the lead time parameter will be set and minimized by adding costs (Wu & Lin, 2004). Priyan and Uthayakumar (2015) developed an inventory model by observing that the quantity of products received from suppliers differs from the quantity of products ordered. The different quantities can be caused by several reasons, such as damaged products, missing products, or human error during calculation. This model also aims to minimize the expected total inventory costs by considering lead time, ordering costs, lost sales, and other relevant inventory costs. The total inventory cost for lead time reduction model by Priyan and Uthayakumar (2015) can be seen in Equation (28).

$$\begin{aligned} EAC(Q, A, \tilde{\beta}, L) &= \theta I(A, \tilde{\beta}) + EAC(Q, L) \\ EAC(Q, A, \tilde{\beta}, L) &= \theta [I_0(A) + I_1(\tilde{\beta})] + \frac{AD}{\alpha Q} + h[k\sigma\sqrt{L} + \tilde{\beta}\sigma\sqrt{L}\psi(k)] \\ &\quad + \frac{h}{2\alpha Q} [\sigma_0^2 + (\sigma_1^2 + \alpha^2)Q^2] + \frac{(\pi + \tilde{\beta}\pi_0)D}{\alpha Q} \sigma\sqrt{L}\psi(k) + \frac{R(L)D}{\alpha Q} \end{aligned} \quad (28)$$

5.5. Model Limitation and Assumption

Note that this optimization procedure can be described as discretized iterative search rather than a closed-form analytical solution. Though useful for decision-support and practical tasks, more formal optimization techniques could be investigated to improve analytical rigor and computational efficiency in the future. As a general rule of thumb in probabilistic inventory theory, the model is developed based on the assumption of normally distributed demand with known parameters, constant (or controllable) lead time, independence between item demands, and stationary demand (no explicit seasonality or promotional effects in demand). These are the assumptions that ensure analytic tractability and a structured cost assessment at the supplier side; to a degree, they are also applicable to the operational environment. For relatively stable FMCG markets with moderate demand variability and steady supplier performance, the assumptions represent a reasonable approximation for tactical decision support. In

contrast, in extremely volatile markets with skewed or intermittent demand, high correlations between items, strong promotional effects, and/or stochastic lead-time variability, additional model expansions would be demanded. Thus the given framework needs to be perceived as a structured model of decision support by having clearly defined boundary conditions rather than a fully dynamic model of complex retail demand systems.

6. Numerical Example

The inventory studied in this case is finished goods inventory from FMCG (Fast Moving Consumer Goods) company in Indonesia. The company needs inventory to ensure the fulfillment of needs and anticipate demand uncertainty. Therefore, the company provides operating stock and safety stock to overcome this uncertainty. The company experienced an increase in sales quantity. However, this increase was also accompanied by an increase in the number of lost sales, averaging 16% per month in a year. In this study, there are 116 types of Beauty & Well-being items. There are 3 suppliers who supply the company with each supplier supplying 58 types of items (supplier 1), 56 types of items (supplier 2), and 2 types of items (supplier 3).

An example calculation for the proposed P multi-item backorder model is illustrated using items N (repairing jelly product) and K (micellar water) from Supplier 1 as representative examples. The complete optimization procedure, however, was performed for all 116 items consisting of 58 items from Supplier 1, 56 items from Supplier 2, and 2 items from Supplier 3. Due to manuscript space limitations, only representative calculation examples are presented in detail. For the items N and K from Supplier 1, assuming that $A_1 = \text{USD } 36,137.14$, $Ch_i = \text{USD } 182,761.92$, $C_{fi} = \text{USD } 838,616$, $D_{N1} = 609.33$ units per year, $D_{K1} = 900.46$ units per year, $L_1 = 3$ months, $\tilde{x}_{a1} = 2.95$, $s_{N1} = 30.49$ units per year, $s_{K1} = 67.11$ units per year, $f(\tilde{x}_a)_1 = 0.0051$, and $\Psi(\tilde{x}_a)_1 = 0.0005$. The calculation steps of inventory policy with the P model multi-item is shown in Table 4.

No.	Step	N (Supplier 1)	K (Supplier 1)
1	Iteration 1		
	Calculate the time between orders (T)	0.01 year	
	Calculate the probability of inventory shortage (α)	0.0016	0.0016
	Calculate the maximum inventory point (R)	21.82 cartons	40.67 cartons
	Calculate the expected value of the amount of inventory shortage (N)	0.01 cartons	0.03 cartons
	Calculate the total cost of inventory	USD 324,225.33	
2	Iteration 2		
	Specifies the time between orders (T)	0.015 year	
	Calculate the probability of inventory shortage (α)	0.0024	0.0024
	Calculate the maximum inventory point	25.19 cartons	46.23 cartons
	Calculate the expected value of the amount of inventory shortage (N)	0.02 cartons	0.05 cartons
	Calculate the total cost of inventory	USD 324,450.66	
3	Iteration 3		
	Specifies the time between orders (T)	0.005 year	
	Calculate the probability of inventory shortage (α)	0.0008	0.0008
	Calculate the maximum inventory point	18.41 cartons	35.01 cartons
	Calculate the expected value of the amount of inventory shortage (N)	0.01 cartons	0.02 cartons
	Calculate the total cost of inventory	USD 324,206.28	
4	Iteration 4		
	Specifies the time between orders (T)	0.0021 year	
	Calculate the probability of inventory shortage (α)	0.0004	0.0004

No.	Step	N (Supplier 1)	K (Supplier 1)
	Calculate the maximum inventory point	16.80 <i>cartons</i>	32.39 <i>cartons</i>
	Calculate the expected value of the amount of inventory shortage (N)	0.0034 <i>cartons</i>	0.01 <i>cartons</i>
	Calculate the total cost of inventory	USD 324,600.84	

Table 4. Example of inventory policy with the *P* multi-item model calculation

Table 4 illustrates the iterative search procedure for determining the ordering interval T_j and the corresponding maximum inventory levels for the proposed *P* multi-item model. The final parameter set is selected based on the minimum expected total inventory cost among the evaluated iterations. In this illustrative calculation, Iteration 3, with $T_j = 0.005$ year, produces the lowest total inventory cost of USD 324,206.28. Therefore, this parameter set is treated as the selected solution for the illustrative calculation, while the final supplier-level results in Table 6 are obtained from the complete optimization procedure across all items.

7. Results and Analysis

The results of the calculations from the developed model are then compared with the reference models mentioned in sub-chapter Perpetual Inventory Model (*Q* model) and sub-chapter Lead Time Reduction Model (lead time reduction model). For the *P* multi-item model and the *Q* model, there is a mixture of shortage costs (backorder and lost sales). Therefore, there are five models whose values will be compared for each supplier to choose the most optimal cost, such as (1) *P* multi-item lost sales, (2) *P* multi-item backorder, (3) *Q* lost sales, (4) *Q* backorder, and (5) *Q* (lead time reduction) by Priyan and Uthayakumar (2015). The selection of optimal policies is the policy that produces the smallest of total inventory costs. First, we will compare the performance of inventory policies for each item from the *Q* backorder, *Q* lost sales, and *Q* (lead time reduction) inventory model by Priyan and Uthayakumar (2015) in each supplier. The results of calculating the minimum total cost of inventory from these three *Q* models for each item from each supplier will be totaled and written as the “selected *Q*”. Table 5 presents representative results of the total inventory cost obtained from the “selected *Q*” policy for Supplier 1, which consists of 58 items ranging from A to BF. The optimization process was conducted for all items supplied by each supplier; however, only partial results are presented to improve readability and maintain manuscript conciseness.

Product Name	Total Inventory Cost (USD)			Total Inventory Cost of the “Selected Q” (USD)
	Q Lost Sales	Q Backorder	Q Priyan and Uthayakumar (2015)	
A	6,844.78	6,844.60	6,841.16	6,841.16
B	3,020.51	3,020.40	3,080.05	3,020.40
C	7,882.16	7,882.24	7,875.13	7,875.13
D	8,851.00	88,507.86	8,882.31	88,507.86
E	7,379.83	7,379.88	7,397.24	7,379.83
F	7,715.84	7,715.55	7,706.74	7,706.74
G	8,581.43	8,581.51	8,585.10	8,581.43
H	16,727.43	16,727.75	16,746.91	16,727.43
I	7,915.32	7,914.98	7,953.39	7,914.98
J	9,400.66	9,400.39	9,391.95	9,391.95
K	5,991.86	5,991.90	5,992.51	5,991.86
L	3,145.05	3,144.83	3,157.90	3,144.83
M	10,575.87	10,575.68	10,570.08	10,570.08
...	...	...	...	...
BE	4,715.09	4,714.86	4,723.85	4,714.86
BF	6,853.33	6,853.04	6,857.43	6,853.04
Total Cost (USD)				326,449.00

Note: The optimization and policy selection procedures were implemented for all items associated with each supplier. Table 5 only displays representative item-level results due to space constraints.

Table 5. Example of total cost inventory of the “Selected Q” from supplier 1

The total cost of the selected Q model from each supplier is then compared with the developed models, P multi-item lost sales and P multi-item backorder, as can be seen in Table 6.

Supplier	Total Inventory Cost (USD)		
	P Multi-Item Lost Sales	P Multi-Item Backorder	Selected Q
Supplier 1	324,206.36	324,205.44	326,449.00
Supplier 2	526,997.98	526,997.16	529,504.81
Supplier 3	8,860.71	8,861.28	8,812.93

Table 6. Summary of the results of the proposed inventory policy

Furthermore, the smallest total cost is also selected for the P model multi-item. A performance comparison based on total inventory cost for each supplier with the P model multi-item is shown in Table 6. The P model multi-item selected for the three suppliers is the P model multi-item with backorders. Even though the total cost of the selected Q model for each item already uses the minimum value, it turns out that the total inventory cost value generated by the P multi-item backorder model is the smallest for Supplier 1 and Supplier 2.

In addition, a performance comparison will be carried out using service levels. This comparison aims to see whether the selected policy solution has the same results as the previous total inventory cost comparison. The performance comparison measures by level of service are similar in steps to comparisons based on total costs. The service level of the Q lost sales, Q backorder, and Q lead time reduction inventory model will be compared first. Once done, the highest service level will be compared with the multi-item adjustment P model. We will look for the

average service level of the three models that produce the highest value. This value will be written as the “selected Q ”. Then, a comparison of service levels for the P model multi-item is also carried out to obtain a solution that produces the highest level of service. The results of performance comparisons by level of service for the P model multi-item are shown in Table 7.

Supplier	Service Level		
	P Multi-Item Lost Sales	P Multi-Item Backorder	Selected Q
Supplier 1	99.90%	99.90%	98.49%
Supplier 2	99.91%	99.91%	98.54%
Supplier 3	99.61%	99.61%	97.29%

Table 7. Comparison results of service levels with the P multi-item and “selected Q ”

The proposed policy solution for Supplier 1 and Supplier 2 is to use the P model multi-item with backorders. These results are in accordance with those shown in Table 6 because the total cost generated from the model is also very small. This contrasts with Supplier 3, which shows a lower level of service when using solutions from the selected Q model. Thus, it is necessary to carry out further performance comparisons of the solutions produced by the selected Q model and the selected P model.

Although total inventory cost minimization constitutes the primary optimization objective of the proposed inventory models, the final policy selection additionally considers implementation feasibility under actual operational conditions. In practical inventory systems, policies with similar quantitative performance may differ substantially in terms of monitoring complexity, system readiness, administrative workload, and responsiveness. Therefore, a secondary managerial evaluation stage using the Point Rating method is introduced after the mathematical optimization process. This stage does not replace the optimization results, but rather supports implementation-oriented policy differentiation among near-optimal alternatives. The criteria considered in the calculation include the total cost of inventory, service level, ease of implementation, the readiness of the information system owned, and the readiness of the human resources to process it. This is in line with a decision-support approach to inventory management, where optimal mathematical solutions are compared to an organization’s constraints prior to implementation.

Criteria	Weight	Q (selected)		P multi-item (selected)	
		Mark	Score	Mark	Score
Total inventory cost	0.25	50.17	12.54	49.83	12.46
Service level	0.3	49.79	14.94	50.21	15.06
Ease of implementation	0.2	42.86	8.57	57.14	11.43
Responsiveness	0.1	42.86	4.29	57.14	5.71
Administration level	0.15	42.86	6.43	57.14	8.57
Total	1		46.77		53.23

Table 8. Calculation results with Point Rating of Supplier 1

Criteria	Weight	Q (selected)		P multi-item (selected)	
		Mark	Score	Mark	Score
Total inventory cost	0.3	37.25	11.17	62.75	18.83
Service level	0.25	49.65	12.41	50.35	12.59
Ease of implementation	0.1	42.86	4.29	57.14	5.71
Responsiveness	0.25	57.14	14.29	42.86	10.71
Administration level	0.1	42.86	4.29	57.14	5.71
Total	1		46.44		53.56

Table 9. Calculation results with Point Rating of Supplier 2

Criteria	Weight	Q (selected)		P multi-item (selected)	
		Mark	Score	Mark	Score
Total inventory cost	0.3	57.42	17.22	42.58	12.78
Service level	0.25	49.44	12.36	50.56	12.64
Ease of implementation	0.1	42.86	4.29	57.14	5.71
Responsiveness	0.25	57.14	14.29	42.86	10.71
Administration level	0.1	42.86	4.29	57.14	5.71
Total	1		52.44		47.56

Table 10. Calculation results with Point Rating of Supplier 3

The incorporation of managerial criteria becomes particularly relevant when competing policies generate relatively close total inventory cost values. As shown in Table 6, the difference between the selected Q policy (USD 326,449.00) and the proposed P multi-item backorder policy (USD 324,205.44) for Supplier 1 is approximately 0.69% of the total inventory cost. This indicates that both policies are quantitatively close in terms of cost performance. Under such conditions, additional managerial considerations such as implementation feasibility, operational responsiveness, administrative complexity, and system readiness become important to support practical policy selection. Therefore, the Point Rating stage is intended to complement the mathematical optimization results rather than replace them.

Based on the results of calculations with the Point Rating, Supplier 3's policy solution also continues to use the selected Q model even though the level of service provided is lower. This is because the total cost of inventory produced is minimal, and the difference in the level of service produced is not that big, which is only about 1.8%. Thus, the policy solution for Supplier 1 and Supplier 2 is to use a multi-item adjustment P model with backorders and a combined Q model (Priyan & Uthayakumar, 2015, and backorder) for Supplier 3.

The proposed optimal policy solution is also better implemented into the SAP system so the company can make decision-making easier. This is done considering that items that use the Q model require continuous checking, or in other words, the Q model will be administered more intensively. The system will provide information about inventory positions, so the company knows when to reorder. The same is done for multi-item P model adjustments. Thus, the company can receive information about the time between orders and the maximum inventory point when the order period is made. Implementing the proposed inventory policy solution has been proven to reduce total inventory costs. Companies can organize items better and more measurably. If a company decides to use the proposed solution, it would be better to prepare more finances to make changes in the inventory system. Thus, companies need to modify the SAP system and conduct workforce training in its operation. The proposed inventory policy model solution can work well if it is supported by suitable tools. Companies also need to pay attention to their relationships with the suppliers concerned. In this paper, the lead time used was assumed to be constant and deterministic. In actual circumstances, the ability of suppliers to

supply their items to companies is not always timely and varies. Companies must evaluate and work with suppliers to determine consistent delivery lead times. Evaluation of relationships with suppliers is seen in terms of time and the accuracy of quantity of items sent so that input in planning and solution calculation activities is more accurate and precise.

The proposed model was tested to determine how well it reflects the real inventory cost structure of the study system. The validation procedure uses the current inventory policy parameters (i.e. existing ordering quantities, reorder points and replenishment intervals) as inputs to the cost functions of the proposed model and compares the total inventory cost obtained with the total inventory cost recorded under the existing policy. The approach validates the representativeness of the cost structure of the model, not the optimality of the proposed solution. Table 11 presents the differences between actual and model-estimated total inventory costs of 0.28%, 0.23% and 1.09% for Supplier 1, Supplier 2 and Supplier 3 respectively. These small differences imply that the proposed cost functions are a good approximation of the real cost behavior of the system with the same replenishment parameters. This confirms that the model is a good representation of the real inventory system before optimization is executed.

Supplier	Total Inventory Cost (USD)		Difference
	Existing Total Inventory Cost (USD)	Total Inventory Cost of The Proposed Model (USD)	
Supplier 1	330,750.46	331,674.05	0.28%
Supplier 2	538,066.46	539,313.33	0.23%
Supplier 3	8,917.72	8,820.49	1.09%

Table 11. Validation results of the proposed model

Comparison of existing and proposed inventory policies is done to see any differences in performance criteria. Comparison of existing and proposed inventory cost components is shown in Table 12 and Table 13.

Supplier	Total Inventory Cost (USD)		Difference
	Existing Total Inventory Cost (USD)	Total Inventory Cost of The Proposed Model (USD)	
Supplier 1	9,569.32	3,024.30	68.40%
Supplier 2	14,731.10	3,661.80	75.14%
Supplier 3	267.32	155.98	41.65%

Table 12. Comparison of existing and proposed inventory cost components

Supplier	Service Level		Difference
	Existing	Proposed Model (USD)	
Supplier 1	95.94%	99.90%	4.13%
Supplier 2	95.36%	99.91%	4.77%
Supplier 3	95.12%	97.29%	2.28%

Table 13. Comparison of existing and proposed model in service level

The proposed inventory policy is able to save total costs consisting of ordering costs, holding costs, and shortage costs with a value of 68.40% for Supplier 1, 75.14% for Supplier 2, and 41.65% for Supplier 3. The magnitude of these cost reductions is explained by several operational mechanisms. First, the main source of savings in ordering and holding costs is the supplier-level coordinated replenishment structure. In the current policy, the items are

replenished independently, and therefore we have frequent unsynchronized replenishment cycles, and the order interval for each item is sub-optimal. The P multi-item model jointly determines a common replenishment interval for all items coming from the same supplier, so reducing the total number of ordering events and spread the fixed ordering cost over a larger combined order, and the maximum inventory level to take into account the real multi-item demand during the protection interval. The coordination effect is strongest for Supplier 1 and Supplier 2, who supply a large number of items (58 and 56 items, respectively) and thus have a higher potential for synchronization gains. The lower saving for Supplier 3 (41.65%) is consistent with this explanation as Supplier 3 supplies only 2 items which limits the benefit of coordination.

Second, introducing probabilistic demand with explicit safety stock calibration reduces the shortage costs compared to the existing policy. The present policy does not seem to take into account demand variability in a systematic way when the parameters for the reorder point are calculated, which results in either excess safety stock or lack of coverage. The model controls the safety stock by the service-level parameter, α , which balances directly the trade-off between the holding cost and the shortage cost under the normally distributed demand. Thus, the enhancement in both cost and service level in all three suppliers (Table 13) happened at the same time.

Third, with respect to generalizability, the magnitude of savings is expected to vary across operational settings according to three factors. These are: (i) the number of items per supplier — the more items the greater the coordination benefit, (ii) the degree of demand variability — the greater the variability the greater the value of explicit probabilistic safety stock calibration, and (iii) the ratio of ordering cost to holding cost — the greater the ordering cost relative to holding cost the greater the benefit of reducing order frequency through coordination. Other organizations with similar characteristics of multiple items from a supplier, probabilistic demand and significant ordering costs should expect similar improvements although the magnitude will depend upon their specific cost parameters.

The service level for the three suppliers increased by an average of 3.73%. The proposed model has the potential to enhance the service level because the expected value of the resulting shortage is very small. This indicates that demand during lead time will be met to a greater extent compared to the existing conditions. The overall results of the sensitivity analysis contribute to understand the robustness and reliability of the proposed inventory policy. The sensitivity of the model to demand, ordering cost and holding cost parameters indicates that the optimal replenishment interval and maximum inventory level are sensitive to changes in these inputs in a meaningful way, which is expected and consistent with standard inventory theory. More importantly, insensitivity to the crashing cost and shortage cost parameters suggests that the optimal policy solution is robust even when these cost estimates have greater uncertainty, which is practically relevant since shortage and crashing costs are often the most difficult parameters to estimate accurately in real operational settings. Regarding reliability, the results are limited by a number of structural assumptions: demand is assumed to be normally distributed, lead times are taken to be deterministic, and the planning horizon is assumed to be stationary. Though the optimal parameter values might vary for cases of highly skewed demand, highly variable lead times or non-stationary planning environment, the proposed framework's supplier-level coordination structure is still valid. The robustness analysis could be extended in future research by examining the performance of the model under different demand distributions or stochastic lead-time environments.

8. Sensitivity Analysis

Sensitivity analysis is performed to see how the optimal solution responds to changes given to a parameter with other parameters unchanged. This analysis aims to enable decision-makers to anticipate changes to the optimal solution if there are changes in parameters. If the optimal solution does not change or only changes slightly, then it can be considered insensitive to the parameter changes. Thus, decision-makers can trust the solution produced by the model. Conversely, decision-makers need to consider optimal solutions sensitive to parameter changes.

Sensitivity analysis will be performed on the selected proposed model for each supplier against the input parameters of each model. The parameters that are subjected to sensitivity analysis are only those that affect the optimality of the solution. Therefore, no sensitivity analysis is performed on changes in purchasing cost parameters. Based on the

results of the proposed policy, the parameters that will be changed are demand parameters, ordering costs, storage costs, crashing costs, and shortage costs.

8.1. Sensitivity Analysis in Demand

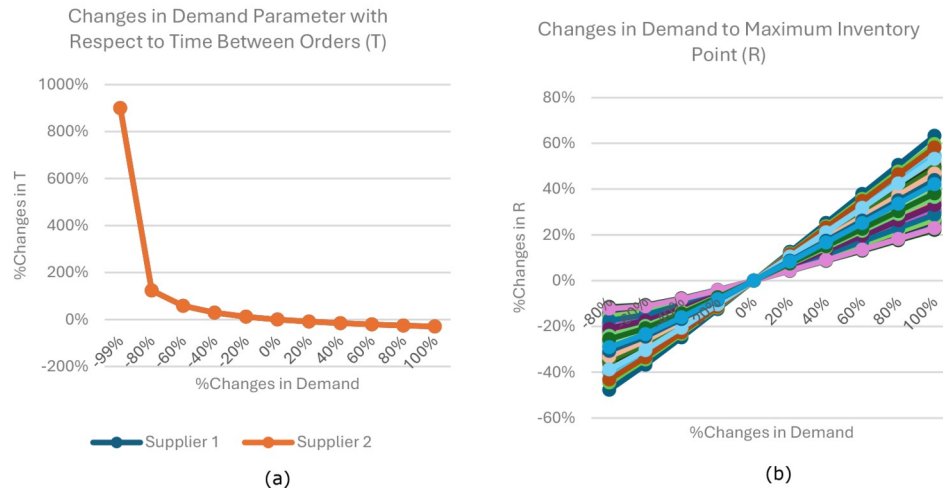


Figure 2. Sensitivity analysis of changes in demand of P multi-item backorder model, with respect to (a) T and (b) R on Supplier 1 and Supplier 2

Based on Figure 2, changes to demand parameters provide a sensitive model solution. An increase in demand decreases the time between orders, making the relationship inversely proportional. This also affects the company's maximum inventory level, which is proportional to the increase in demand for items. Furthermore, sensitivity analyses of the demand parameters on Supplier 3's items were performed.

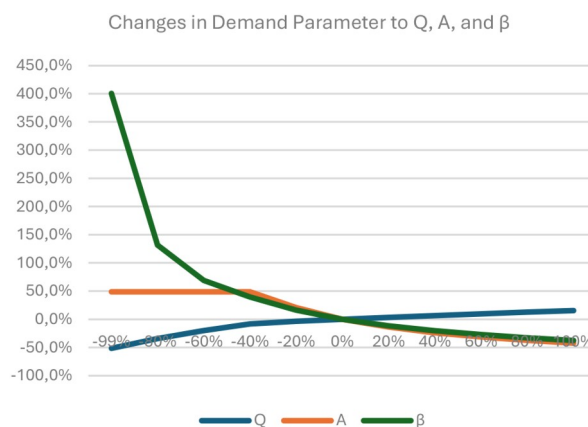


Figure 3. Sensitivity analysis of changes in demand of Q Priyan and Uthayakumar (2015) model, with respect to Q , A , and β on Supplier 3

Based on Figure 3, the increase in the demand parameter value is proportional to the increase in the order lot size. A different thing is shown in the change in the value of A and β . The greater the demand value, the more likely the model is to reduce the order cost, which is indicated by the decrease in the order cost value and the fraction value β . This change means that the model used is sensitive to changes in demand parameters.

8.2. Sensitivity Analysis in Ordering Cost

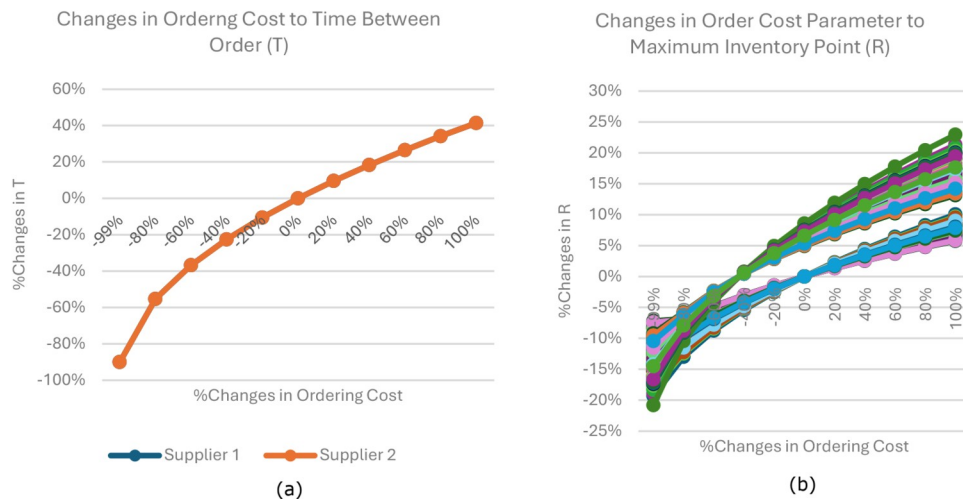


Figure 4. Sensitivity analysis of changes in ordering cost of P multi-item backorder model, with respect to (a) T and (b) R on Supplier 1 and Supplier 2

Figure 4 shows that the change in the ordering cost parameter is directly proportional to the inter-order time and also the maximum inventory point. The increase in ordering cost results in an increase in the inter-order time to reduce the frequency of orders, leading to an increase in the maximum inventory level. Thus, the multi-item P adjustment model with backorder is sensitive to changes in the ordering cost parameter. Furthermore, sensitivity analysis was also conducted to the changes in ordering cost parameter for the Q backorder model on Supplier 3. The result shows that the model is sensitive to changes in ordering cost.

8.3. Sensitivity Analysis in Holding, Crashing, and Shortage Costs

The following parameters were selected for sensitivity analysis: holding cost, crashing cost, and shortage cost. The results show that for both models, P multi-item and Q models (Priyan & Uthayakumar, 2015, and backorder), the proposed model is sensitive to changes in holding cost and not sensitive to changes in crashing cost and shortage cost.

9. Conclusions

This study is based on several simplifying assumptions, such as normally distributed demand, stationary parameters, independent item demands, and lead times that are constant or controllable. Although these assumptions align with classical inventory theory and allow for analytical tractability, future research could expand the framework to include correlated demand structures, seasonal patterns, and dynamic promotional effects for increased realism in highly volatile FMCG environments.

Several normal-distributed probabilistic inventory policy models are proposed using perpetual inventory model (Q) and periodic inventory model (P). Furthermore, we considered multi-item single-source (MISS) to integrate with existing models because of the potential for items to be supplied by the same supplier. There are four models that are utilized in this paper, i.e. P -model, Q -model, multi-item model, and lead time reduction model. In addition, we also investigate the periodic and continuous/perpetual model with varying mixture shortage costs (backorder and lost sales).

From the above discussion, the extension of the inventory model to improve and optimize service levels and minimize total inventory costs while incorporating service performance and implementation feasibility consideration is an appropriate approach. Some analytical results have been discussed to find the proposed model's optimal solution. Based on four models, we use several decision variables to compare each method for optimal policies. An easy-to-apply solution procedure is provided to derive the optimal solutions. Numerical examples are given to illustrate our results.

Practical insights for supply chain managers emerge from the numerical results, which demonstrate significant cost savings across different suppliers. The proposed model (P multi-item) excelled in two suppliers, while the Q lead time reduction model by Priyan and Uthayakumar (2015) and Q backorder is optimal for Supplier 3. The proposed inventory policy is able to save total costs consisting of ordering costs, holding costs, and shortage costs with a value of 68.40% for Supplier 1, 75.14% for Supplier 2, and 41.65% for Supplier 3. The research results also increased the service level with final service level 99.90% for Supplier 1, 99.91% for Supplier 2, and 97.29% for Supplier 3.

This study offers some insights from the numerical results for stakeholders to respond to disruptions in inventory policy to overcome lost sales issues. The stakeholders can consider some critical decisions such as (i) the proposed inventory policy can be implemented by companies as an effort to reduce total inventory costs and improve service levels, (ii) it is necessary to supervise and evaluate suppliers so that demand fulfilment can be guaranteed (iii) companies need to integrate the proposed model solution with the information system used today.

For further research, this study can be extended in several ways, such as (i) develop models for demand characteristics with distribution patterns other than normal distributions, (ii) consider price changes that could have a significant impact on inventory cost, (iii) conduct a joint review of the company condition with suppliers so that the calculation of inventory policy can be mutually beneficial, and (iv) create an information system to provide knowledge about inventory positions so that companies can integrate directly with the built system.

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Authors' contributions

Yosi Agustina Hidayat: Conceptualisation, Methodology, Data Curation, Supervision, Formal Analysis, Funding Acquisition, Writing – Original Draft, Review & Editing.

Nadin Fadilla: Conceptualisation, Data Curation, Visualisation, Methodology, Writing – Original Draft, Writing – Review & Editing.

Data availability

Data available upon request.

Use of Artificial Intelligence

The authors declare that they wrote this article without using artificial intelligence for the writing of the text, data analysis, the creation of graphs, or any of the other activities carried out. The authors used Grammarly to improve the language and take full responsibility for the content of the publication.

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